university course so as consistently to encourage deep approaches to learning in as many students as possible (Baeten et al. 2010). And that depends on being able to understand the nature of academic understanding within the course and to describe, and appropriately support, the specific learning processes involved in a deep approach to learning in that subject area.

Cross-References

▶ Attitudes and Learning Styles
▶ Cognitive and Affective Learning Strategies
▶ Learning About Learning
▶ Learning and Understanding
▶ Learning Strategies
▶ Learning Style(s)
▶ Perceptions of the Learning Context and Learning Outcomes
▶ Phenomenography
▶ Self-regulated Learning

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Approaches, Inquiries, and Paradigms in Music Education Research

▶ Research Methods in Music Instruction and Learning

Approach-Withdrawal Motivation

▶ Approach and Avoidance Motivation

Appropriability

▶ Absorptive Capacity and Organizational Learning

Appropriation

▶ Internalization

Approximate Dynamic Programming

▶ Reinforcement Learning in Animals

Approximate Learning

▶ Approximate Learning of Dynamic Models/Systems

Approximate Learning of Dynamic Models/Systems

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Synonyms

Approximate learning; Asymptotic performance; Dynamical system

Definition

Dynamical systems model the evolution of a system with unknown parameters. The goal of a learning procedure is to estimate the parameters of the system, possibly from a set of known examples, such that the system behavior on future inputs is accurately predicted. Such a learning procedure typically uses methodologies from techniques in statistics and computer science and can be computationally intractable for some systems.

Theoretical Background

A dynamical system models the state-space evolution of a system. In the discussion below, we refer to free parameter that models the change in dynamics by "time." Many versions of such systems are possible, depending on whether the state variables are continuous or discrete (quantitative), the time variables are continuous (e.g., partial differential equation, delay equations) or discrete (e.g., difference equations, quantized descriptions of continuous variables) and whether the model is deterministic or probabilistic in nature. In addition, such a model can also be hybrid in nature in the sense that it may combine continuous and discrete timescales and/or continuous and discrete time variables. For example, the dynamics of a discrete-time continuous-state system with a single output can be written down as

$$\begin{aligned} x_i(t+1) &= f_i(x_1(t), x_2(t), \cdots, x_n(t), z_1(t), z_2(t), \cdots, z_m(t)), \\ & \quad i = 1, 2, \cdots, n \\ y(t+1) &= h(x_1(t+1), x_2(t+1), \cdots, x_n(t+1)) \end{aligned}$$

where $x_1, x_2, \ldots, x_n$ are the state variables, $z_1, z_2, \ldots, z_m$ are the m variables representing inputs to the system, y is the output variable that provides information about measurable performance of the system, t is the time variable governing the dynamics and f_i's and h are real-valued functions with unknown parameters (also called weights) characterizing the nature of the dynamics. For example, the function f_i's could be the so-called sigmoidal function:

$$\begin{aligned} & f_i(x_1(t), x_2(t), \cdots, x_n(t), z_1(t), z_2(t), \cdots, z_m(t)) \\ &= \left(1 + e^{-\left(\sum_{i=1}^{n} \theta_i x_i(t) + \sum_{j=1}^{m} \theta'_j z_j(t)\right)}\right)^{-1}, \end{aligned}$$

where $\theta_1, \theta_2, \ldots, \theta_n$ and $\theta'_1, \theta'_2, \ldots, \theta'_m$ are the unknown real-valued parameters and $\boldsymbol{e}$ is the base of natural logarithm. We will use Θ to denote the vector of unknown parameters.

In a typical learning scenario, we have an unknown function $g(x_1, x_2, \ldots, x_n)$ that we would like our system to compute. We "train" our system by providing a set of inputs, drawn from a probability distribution, with their corresponding value of the g function, to the system, say one at a time, for a finite time period t_0. The goal is to efficiently compute the parameters in Θ such that the generalization error, namely the expected error in the output of the system to the true output that we desire for the next input drawn from the same distribution, is minimized (or, within a desired bound).

For further details on dynamical systems see standard textbooks such as Sontag (1998) and for further details on basic learning theory see Kearns and Vazirani (1994). For some interesting applications of dynamical systems to systems biology see the excellent survey paper Sontag (2005).

Important Scientific Research and Open Questions

Dynamical systems exhibit a fascinating interplay between several areas such as biology, control theory, discrete mathematics, and computer science, and have a wide range of applications in modeling and simulation in many diverse areas such as modeling biological processes, in quantum computing, in self-assembly problems in nanotechnology applications and social networks. Broad scientific investigations in modeling of dynamical systems include difference in convergence to steady states, effect of feedback loops on stability and dynamics, robustness in presence of noise, etc. Furthermore, there are interesting special subclasses of dynamical systems, such as piecewise-linear systems and monotone systems, that have been of considerable interest in recent times especially due to their applications in systems biology but are still far from being completely understood.

There are several directions of research associated with the training and computational capabilities of dynamical systems; below we outline several directions.

One direction of research deals with the computational capabilities of such dynamical systems, typically in specific settings such as artificial neural network models, assuming that the number of state variables is unlimited. This type of research can be traced back to its origin to the old work of the famous mathematician Kolmogorov (1957) who essentially provided the first (nonconstructive) result on the representation capabilities of simple types of dynamical systems obtained by superposition of a set of basis functions. This type of research ignores the training question itself, asking instead if it is at all possible to exactly or approximately compute arbitrary or interesting classes of functions. Many of the results and proofs in this direction are existential only and serve to provide the limiting computational capabilities of dynamical systems.

Another direction of research in learnability of dynamical systems takes an approximation theoretic point of view. This direction overlooks the parameter estimation phase in learning and instead is concerned with bounding the overall error if the best possible parameters with a given system architecture were to be eventually found. An example of such results is Barron (1991).

The third direction research deals with is related more closely to the training phase of learning problems via the so-called sample complexity questions that attempts to quantify the amount of information (number of examples) needed in order to characterize a given unknown input-output mapping. An important technical development in this area culminated in deriving information-theoretic bounds for sample complexities via VC-dimensions (Vapnik 1982) and their suitable extensions to real-valued computations.

A fourth research perspective in approaching theoretical questions regarding learning lies in investigating, for a given architecture of the dynamical system, if there exists a fundamental barrier to training, namely a barrier that is insurmountable no matter which particular parameter estimation algorithm one uses. This line of research was motivated by a frequent observation that many parameter estimation algorithms often runs very slowly for high-dimensional data and is frequently referred to as the "curse of dimensionality." Of course, if we are allowed to adapt the architecture of the dynamical system to the data such as in incremental learning techniques, then we would not be subject to such a barrier.

Cross-References

► Connectionist Theories of Learning
► Formal Learning Theory
► Hierarchical-Network Model for Memory and Learning
► Learning in Artificial Neural Networks
► Mathematical Models/Theories of Learning
► PAC Learning
► Probability Theory in Machine Learning
► Supervised Learning

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Approximate Number System

► Accounting and Arithmetic Competence in Animals

Approximative

► Approximative Learning Vs. Inductive Learning

Approximative Learning Vs. Inductive Learning

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Synonyms

Approximative; Learning