LIQUIDITY AND RESILIENCE OF CREDIT NETWORK: A SIMULATION STUDY

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ABSTRACT

We consider a model of credit network introduced in [1], and extend the analysis to account for properties empirically observed in real credit network, notably degree-preference transaction regime, risk-averse and external shocks. Trade-off between transaction path length and capacity of path is also studied. It is shown that credit network does not need to be very dense to obtain high liquidity level. Network size has no effect on the success probability for BA graphs. Credit network is robust, especially with respect to the condition of edges failure in ER graph. Transactions with degree-preference regime will perform better liquidity level than transactions with uniform regime at low network density. We find that risk of transactions will tend to become stable when average degree increases. Success probability is higher in the case of minimum-capacity transaction than it is for shortest-path transaction at low network density.

KEYWORDS

Credit network, Liquidity, Resilience, Degree-Preference, Risk-Averse

1. INTRODUCTION

Credit network represents a way to model trust between entities. For example, banking system can be modelled as banks exchanging their own currencies depending on the trust to their counterparties. There are also many credit networks transact with non-government issued currencies which known as *scrip*, for example Bitcoin and Facebook credits. One of the key problems is that of understanding the steady properties of credit network over a long period of time. It is important to focus on to what extent the liquidity level depends on the network structure? How the resilience of credit network is to against external shocks? And how to ensure the liquidity of credit network without devaluing the currency?

Recently a growing part of literatures in this field have been devoted to model credit network, Dandekar et al., DeFigueiredo and Barr, Wellman and Wiedenbeck, Ghosh et al. and Karlan et al. [1, 2, 3, 4, 5] have established many alternative models of trust or credit. Here we consider the credit model introduced by Dandekar et al. in [1]. In their model entities in credit network can transact with each

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other for certain amount payment in their respective currencies. It means that no common currency is needed as long as there always exists a chain of sufficient credit from payee to payer. These models are designed to conduct analytical solutions so that omit a lot of characteristics observed in reality. Despite there are many researches on network formation and Sybil attacks of credit network [6, 7], they do not concentrate on the combination of theoretical model and practical considerations comprehensively like our paper.

In this paper we extend the analysis to account for properties empirically observed in real credit network, notably degree-preference transaction regime, transaction threshold and external shocks. We also study the trade-off between transaction path length and capacity of path. Our findings with simulation methods can not only suggest some directions for future research but also pave a path to theoretical study on credit network. The rest of the paper is organized as follows, in Section 2 we review the model introduced in [1], in Section 3 we present the results of simulations with different settings. Finally in Section 4 we provide the conclusion.

2. THE MODEL

In this section we describe the model introduced in [1] and review its main features. A generalized credit network G= (V, E) is a simple graph with n nodes and m edges. Nodes represent entities i.e. financial institutions, edges represent the credit between a pair of nodes which is directed and weighted. Edge (u, v) with weight C_{uv} implies that node u has extended a credit line of C_{uv} units to v in v's currency. As an example in Figure 1, two nodes (A and C) are selected and we route p unit payment from A to C along the shortest path. The transactio9n is successful as long as there exists a directed path from A to C and the capacities of edges on the path are larger than p. As a result of the transaction, the capacity of edges on the shortest path will become $C_1 - p$ and $C_2 - p$ a (since A and B have depleted part of their credit line). And the capacities of reversed edges on the shortest path become to p which means that B and C can trust for A and B for up to p units of its own currency. .

We are interested in understanding how the success probability over a long period of time performs depending on different properties of credit network.

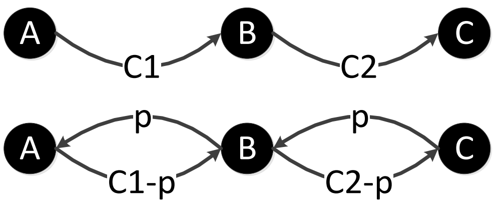

Figure 1: an example of credit network with three nodes

In [1], numerical results are presented for the case of graphs with average degree varies, graphs with capacity varies and effect of variation in network size. In this paper we intend to study credit network with degree-preference transaction regime, transaction threshold and external shocks which are often observed in real world. We also consider the trade-off between transaction path length and capacity of path.

3. SIMULATIONS AND RESULTS

The model is studied numerically for different values of parameters in this section. The goal of simulations is to understand how different parameters affect the stead-state success probability. The results are presented from two stylized networks: Erdos-Renyi random graph $G_{ER}(n,p)$ and Barabasi-Albert scale-free graph $G_{BA}(n,m)$[8, 9, 10]. The parameters of interest are the number of nodes (n), credit capacity (c), the probability to add an edge between two nodes in ER graph (p), the number of each new added node creates with old nodes (d). We do transactions 1000 time steps and compute the stead-state success probability which is the number of success transactions divided by the total transaction time steps. Simulations are run on 100 graphs with same parameters and average success probability are finally achieved.

3.1 Liquidity of Credit Network

3.1.1 Network Properties and Liquidity Level

In complex network theory it is clear that network structure will affect the performance of the system. Empirical evidences show most real networks have scale-free topologies. So a nature question is that if scale-free topology have better liquidity level than others, and how liquidity of credit network will be with different topologies? To test this issue, we initial credit networks with different topologies and conduct transactions between two nodes chosen randomly.

We consider a network consisting of N=100 entities, capacity=3. The parameter p in ER graph varies from 0.2 to 0.8, we pick d in BA graph corresponding to each p such that $2d \approx (n-1)p$ so that we can compare the liquidity of two networks.

Results are reported in Figure 2. Both graphs have concave no-decreasing success probability with average degree varied and ER graph performs better liquidity level than BA graph. It is also noticed that the success probability of both graphs will reach more than 0.85 when the average degree as low as 60 which means the network does not need to be very dense to obtain high liquidity level.

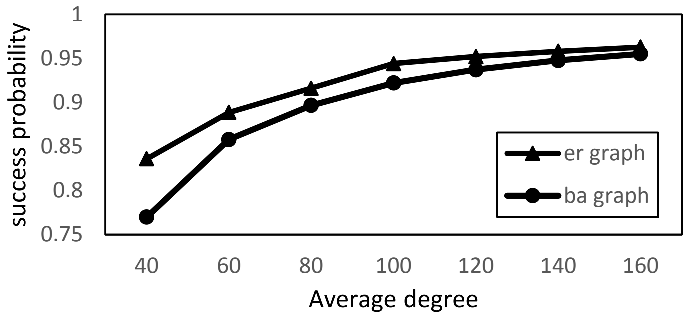

Figure 2: comparison of liquidity level on ER graph and BA graph with average degree varies

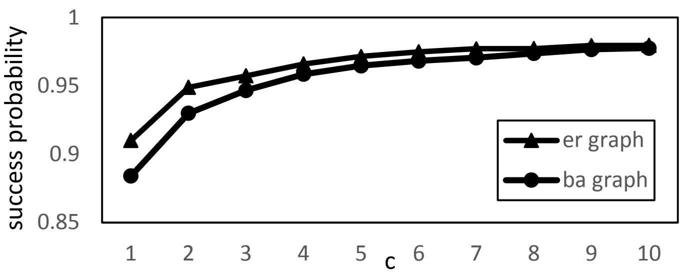

Figure 3: comparison of liquidity level on ER graph and BA graph with capacity varies

To determine the effect of variation in credit capacity on liquidity level, we initial graphs with N=100, p=0.1, d=5 and set graphs with c varying from 1 to 10. As we can see from Figure 3, both graphs have concave no-decreasing success probability with credit capacity varies. And ER graph still performs better liquidity level than BA graph.

We also consider the effect of network size on liquidity level. From Figure 4, the case of BA graph shows network size has no effect on the success probability and liquidity of ER graph will increase with growth of number of nodes. It is intuitively understood that probability of successful transactions in ER graph will increase with more links, but with growth of network size, large number of nodes in BA graph still have low degree which may leads to steady ratio of successful transactions.

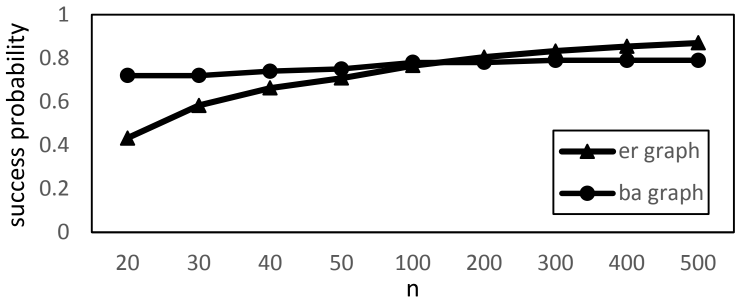

Figure 4: the effect of network size on liquidity level

3.1.2 Resilience and Liquidity Level

In this section we focus on resilience of credit network. Resilience of network means network will keep a relatively high liquidity when there are external shocks. In our model, external shock can be represented as removing certain amount of nodes or edges in the credit network. On the one hand, nodes hit by the shock will lead to bankrcies and will be removed with their adjacent edges, on the other hand, if shock hit the trust relationship between two nodes, the only edge hit will be removed.

Firstly we consider to remove *j* proportion of all edges of graph which chosen randomly to determine how the success probability would change comparing with the normal condition. We then remove *k* proportion of all nodes and their adjacent edges. The parameter *j* and *k* equals to 0.1, 0.3, and 0.6 respectively.

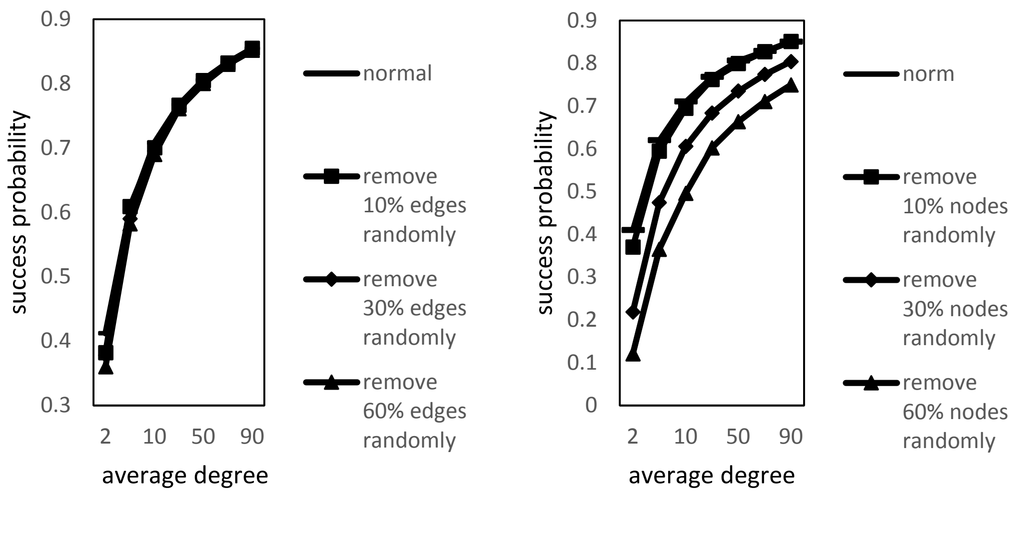

Figure 5: liquidity of ER network with shock on certain amount of nodes and edges

From Figure 5 and 6 it is shown that on ER graph removing more proportion nodes decreases the success probability of transactions. Comparatively, removing the same ratio of edges has no significant effect on liquidity level. For BA graph, it is shown that removing more proportion edges and nodes decrease the success probability of transactions. Compare with ER graph, liquidity level of BA graph decreases more sharply.

The fact from the simulation indicates that credit network is more robust, especially with respect to the condition of edges failure in ER graph. In terms of characteristics of credit network, one node's default has only local influence, and the loss will not spread to other nodes who don't have trust relationship with the failed node. In ER graph every node are equal important, so it will not decrease the success probability significantly if we remove some edges between nodes.

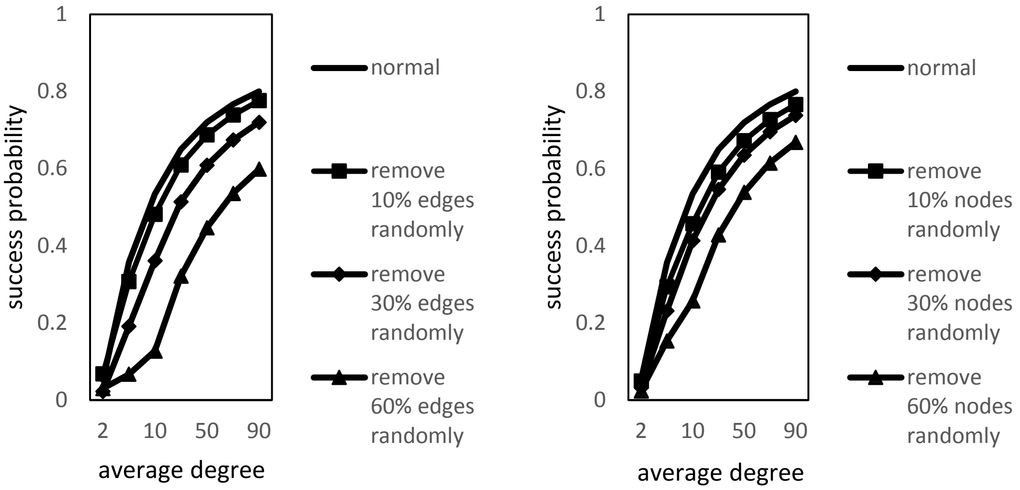

Figure 6: liquidity of BA network with shock on certain amount of nodes and edges

3.2 Degree-Preference and Risk-Averse

Empirical evidences show that entities such as financial institutions will prefer to transact with others which are more reliable, and will be inclined to risk-averse during transactions. In this section, we will compare two transaction regimes uniform and degree-preference to discuss how the transaction regimes affect the success probability of credit network over a long period of time. We also consider how the liquidity level varies if transaction threshold exists.

We explore the case of BA graph while fixing other parameters with d varies from 5 to 90. The counterparty node is selected uniformly under uniform transaction regime, and we select counterparty node depends on its in-degree which represent the ability of being trusted under degree-preference transaction regime.

Results are reported in Figure 7. It is shown that transactions with degree-preference regime will perform better liquidity level than transactions with uniform regime at low network density, but these two will become equal with network density increased. The conclusion indicates that it is a better choice to use degree-preference regime to do transactions when network is not very dense.

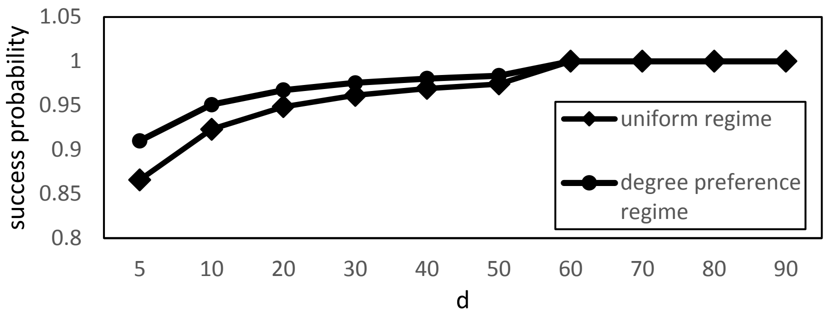

Figure 7: comparison of liquidity level under different transaction regime

Previously we have considered nodes would transact with degree-preference. Another important feature is risk-averse which means nodes would avoid risk during transactions. Hence we introduce "average credit of transaction path" (ACP) into the model. ACP represents the total in-degree of nodes on the transaction path divided by the number of nodes on the path.

$$ACP = \sum_{j \in d(v)} w_{in}^{j} / \#d(v)$$

In the equation, d (v) is the set of nodes which are on the transaction path.

On the other hand, we compute the average in-degree of nodes in the network (ACN).

$$ACN = \sum_{j \in G(v)} w_{in}^{j} / \#G(v)$$

So we can consider an extended threshold model. If ACP>=ACN, it means the transaction is successful and if ACP<ACN, the transaction will be counted as failed.

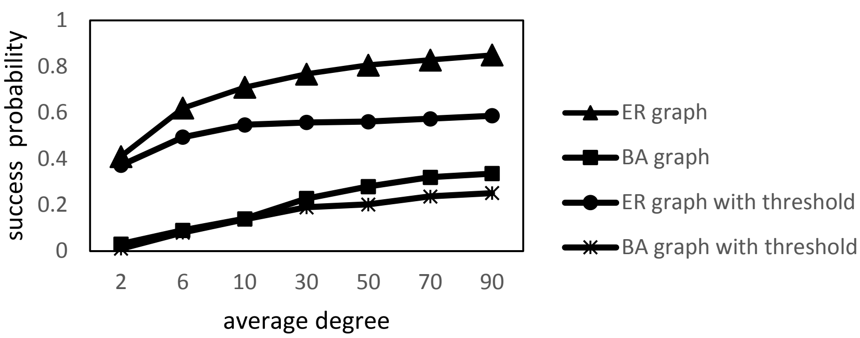

Figure 8: comparison of liquidity level with threshold and without threshold

The result indicates that there is no clear difference between two curves in ER and BA graph when average degree is low. But when average degree grows, graphs with threshold have less success probability than graphs without threshold. We also notice that the area between two curves can be seen as the risk of transactions. It is clear that risk of transactions will tend to become stable when average degree increases, which means increasing network density can effectively control the risk of transactions.

3.3 Criteria of transaction path selection

An important hypothesis in [1] is that they assume nodes transact along the shortest path between them, but in reality, financial institutions will not only consider the length of transaction path but also the capacity of the path. In this section, we study the issue how two nodes would select feasible transaction path and analyze the trade-off between different parameters.

3.3.1 Shortest-Path and Trade-off

A nature question is how the success probability would perform if the cutoff of transaction path is fixed? We conduct simulations on ER graph to compute success probability with cutoff increases under different scenarios. We set benchmark with N=200 and p=0.1, then increase N (N=300, p=0.1) and p (N=200, p=0.2) separately, finally we vary N and p with network density fixed (N=100, p=0.2).

Table 1 Success probability with cutoff increases under different scenarios

Benchmark	N=200	cutoff=1	cutoff=2	cutoff=3	cutoff=4
	p=0.1	**0.056**	**0.386**	**0.972**	**0.998**
Increase N	N=300	cutoff=1	cutoff=2	cutoff=3	cutoff=4
	p=0.1	**0.058**	**0.536**	**0.996**	**1.0**
Increase p	N=200	cutoff=1	cutoff=2	cutoff=3	cutoff=4
	p=0.2	**0.104**	**0.88**	**1.0**	**-**
Fix network density	N=100	cutoff=1	cutoff=2	cutoff=3	cutoff=4
	p=0.2	**0.09**	**0.6**	**0.99**	**0.994**

We compute the liquidity level with cutoff increases under different scenarios. The results in Table 1 show that *p* influences the success probability stronger than *N* under different cutoffs. It has some practical meanings that increasing *p* is more efficient than increasing number of nodes when length of transaction path is constrained.

3.3.2 Minimum-Capacity

In reality, because of the electronic payment system, the length of transaction is not important than before. Financial institutions prefer to find transaction path with minimum capacity to save cost.

We are trying to find minimum-capacity of path to do transactions in ER graph and compare the liquidity level with shortest-path, set N=100, p=0.3 and capacity varies from 1 to 30.

From Figure 9 we can see that success probability is higher in the case of minimum-capacity transaction than it is for shortest-path transaction. While we increase the capacity of edges, both two cases are convergent with success probability at 0.95. So that means if credit network is established with low credit capacity, we had better do transactions with minimum-capacity criterion.

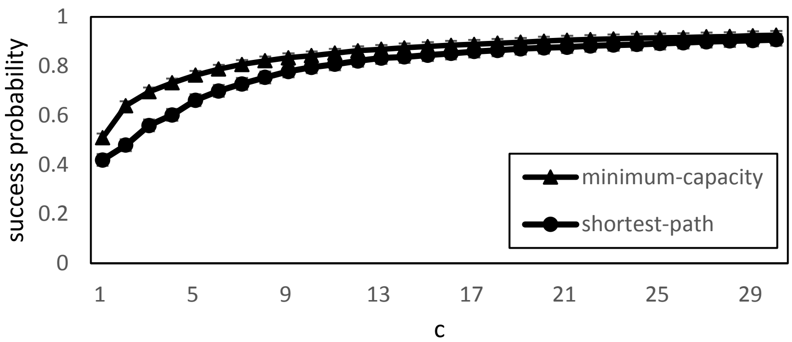

Figure 9: comparison of liquidity level with different transaction path selection

4. CONCLUSION

In this paper we have studied the model introduced in [1] with some properties of credit network empirically observed in real world, we consider the transaction under degree-preference regime, transaction threshold and external shocks. We also study the trade-off between transaction path length and capacity.

In our numerical simulations we observe that credit network does not need to be very dense to obtain high liquidity level, in particular the case of BA graph shows network size has no effect on the success probability. For BA graph, removing more proportion edges and nodes decrease the success probability of transactions. Transactions with degree-preference regime will perform better liquidity level than transactions with uniform regime at low network density, we found that risk of transactions will tend to become stable when average degree increases, which means increasing network density can effectively control the risk of transactions. Finally, we found that parameter p influences the success probability stronger than parameter N with different cutoffs in ER graph. And success probability is higher in the case of minimum-capacity transaction than it is for shortest-path transaction at low network density.

The model and results above suggest some directions for future research. Some of the assumptions should be loose to fit the real credit network, for example allow for transaction fee or exchange rate in the network. And nodes characteristics should be include in the model to calibrate the result and it may provide potentially valuable insights.

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