Identification of Time Varying Systems

Erwin R. Boer and Robert V. Kenyon, *Member IEEE*
Department EECS (m/c 154), University of Illinois at Chicago, BOX 4348, Chicago, IL 60680

Abstract—**An algorithm that uses an optimal control theoretic approach to identifying time varying systems is presented. The hybrid cost function used by this algorithm removes many of the limitation associated with identification methods based on the familiar error based cost function. This results in improved parameter tracking of a wide variety of systems. Of particular interest is identification of systems and their adaptation mechanisms observed in man-machine applications.**

I. Introduction

Recursive Least Squares (RLS) or the Kalman Filter system identifiers have been used frequently on time varying systems [3]. The underlying statistical approach used in the RLS produces three important deficiencies: biased parameter estimates when using forgetting factors less than 1, varying alertness to system parameter changes, and parameter drift in case of non-persistent excitation [4].

The new method presented here is based on optimal control theory and not on statistics. By explicitly modeling first order parameter variations and adding a cost on these parameter variations to the error based cost function results in a hybrid cost function whose counterpart lies in optimal control theory. This algorithm gives unbiased estimates, has a constant level of alertness and shows no drift in case of non-persistent excitation.

II. Theory

Linear MIMO (multiple input multiple output) systems are generally modeled as:

$$y_n = \theta_n^T \phi_n + v_n, \quad (1)$$
$$\theta_{n+1} = \theta_n + w_n, \quad (2)$$

where v_n and w_n are noise samples, θ_n is the system parameter vector, and ϕ_n a vector consisting of past output samples and past and present input samples [3].

The new identifier which we coined SIOC (System Identification via Optimal Control) extends 2 to:

$$\theta_{n+1} = \theta_n + u_n + w_n, \quad (3)$$

where $\{u_n\}$ represents the first order time variations in $\{\theta_n\}$ and w_n as above. Taking u_n and w_n together and calling it δ_n gives the state equation for SIOC:

$$\theta_{n+1} = \theta_n + \delta_n, \quad (4)$$

Note that $\{\delta_n\}$ in 4 represents a stochastic process with time varying statistics.

Almost all currently available identification methods are based on minimizing the following error cost function:

$$J(\{v_n\}) = \sum_{n=0}^{N} v_n^T v_n, \quad (5)$$

in which $v_n = y_n - \theta_n^T \phi_n$. Note that v_n represents the estimation error in 5, whereas it represents noise in 1; these two representations are identical when θ_n is identified with zero error.

SIOC's hybrid cost function is:

$$J = \frac{1}{2} \sum_{n=0}^{N-1} \left[v_n Q_n v_n + \delta_n^T R_n \delta_n \right] + \frac{1}{2} v_N Q_N v_N \quad (6)$$

where Q_n and R_n are weighting matrices and J is a function of $\{v_n\}$ and $\{\delta_n\}$. Equation 6 is called hybrid because it minimizes the weighted powers of two physically different components: signal vector v_n and a parameter vector δ_n. Note that setting R_n to the zero matrix and Q_n to the identity matrix in 6 results in 5.

Comparing the system model in 1 and 4 with the conventional state control model [1, 2] shows that θ_n represents the state, y_n the output of the system, ϕ_n the system itself and δ_n the control vector. Furthermore, SIOC calculates a parameter vector that minimizes the difference between the model output and the observed system output similar to the way optimal control theory calculates a control vector that minimizes the difference between the system output and a given track. Since SIOC and optimal control theorety are similar in structure, many aspects such as controllability, stability and observability carry over.

The identification problem can now be formulated as finding the time series $\{\delta_n\}$ that minimizes 6. The minimization of 6 applies the theory of Lagrange multipliers whereby 1 and 4 are the two constraint equations. The solution can be obtained via the sweep method which entails

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backward and forward evaluation of a series of difference equations [1, 2]. The SIOC identifier algorithm is:

$$\begin{aligned} S_N &= \phi_N Q_N \phi_N^T \\ v_N &= \phi_N Q_N y_N \\ S_n &= \left[I - [S_{n+1} + R_n]^{-1}\right] S_{n+1} + \phi_n Q_n \phi_n^T \\ v_n &= \left[I - [S_{n+1} + R_n]^{-1}\right] v_{n+1} + \phi_n Q_n y_n \\ \theta_0 &= \text{given} \\ \theta_{n+1} &= \theta_n + \delta_n \\ \delta_n &= [S_{n+1} + R_n]^{-1} [v_{n+1} - S_{n+1}\theta_n] \end{aligned} \tag{7}$$

Since R_n and S_n are both positive definite matrices, the bracketed matrix has an inverse. Note also that SIOC is a batch analysis method but gives a parameter estimate at each time step and not an average as in most batch identification methods. Because SIOC does not make any assumptions about causality it can also be applied to non-causal systems, an application that is beyond the scope of most identification methods.

To provide an intuitive feel for 7, two extreme cases are considered. First, setting R_n to the identity matrix with infinitesimal positive diagonal entries (to assure that R_n remains positive definite) removes the cost on δ_n, hence allowing for any parameter change. In the limit, the expression for θ_{n+1} in 7 reduces then to:

$$\theta_{n+1} = \left[\phi_{n+1} Q_{n+1} \phi_{n+1}^T\right]^{-1} \phi_{n+1} Q_{n+1} y_{n+1} \tag{8}$$

which is the instantaneous least squares estimate. Second, setting R_n in the last equation of 7 to the infinite identity matrix shows that δ_n becomes zero and therfore θ_n equals the given θ_0 for all n.

Replacing R_n in 6 by $(\phi_n K_n \phi_n^T)^{-1}$, optimally incorporates the idea that strong signals contain more information than signal almost buried in noise (persistence of excitation). In other words, the cost on large δ_n is small when the SNR is large and vise versa.

III. Application

To demonstrate SIOC's performance in tracking time varying system parameters, an ARMA model with a second order Auto Regressive and a first order Moving Average (ARMA) polynomials in z was simulated ($y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 u_n + b_1 u_{n-1} + v_n$), whereby a_1, a_2, b_0 and b_1 parameters varied sinusoidally (thin solid line in Fig. 1 shows a_2). The input and noise signals u_n and v_n were zero mean Gaussian white processes with a SNR of 0 dB. The weighting matrices for SIOC were: $Q_n = I$ and $R_n = 900I$ for all n, and $\theta_0 = 0$. The RLS algorithm as in [3] with a forgetting factor of 0.985 and an initial error covariance matrix of $10,000I$ was used in the comparison. These settings were chosen such that both algorithms performed most similar.

Tracking performance of parameter a_2 for both identifier is shown in Fig. 1. This figure demonstrates SIOC's better parameter tracking performance compared with RLS. The RLS algorithm quickly looses track of the time varying parameter trajectory whereas SIOC exhibits good average parameter tracking. Even though RLS becomes less and less alert as time passes, SIOC show no degradation in alertness. The main reason for the RLS's inferior tracking of the predefined parameter trajectories lies the fact that parameter time variation are not explicitly incorporated in its algorithm.

Due to SIOC's ability to track time varying system parameters, identifying adaptive controllers of which the human is a prime example, becomes easier. Currently SIOC is being applied to human preview control to explore adaptation mechanisms associated with controlling under time varying preview, system, or visual conditions.

References

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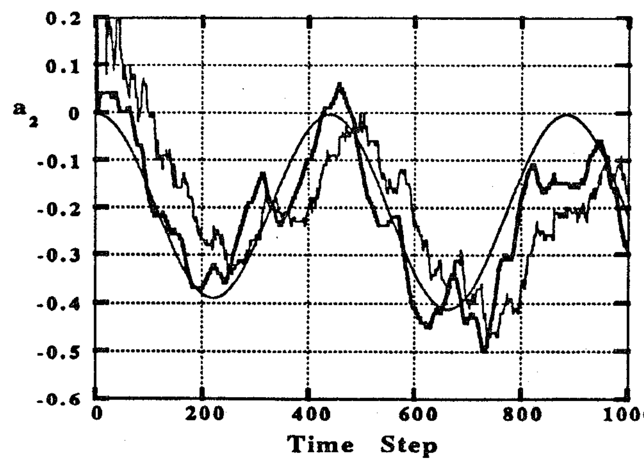

Figure 1: True system parameter a_2 (thin) and their identified estimates (SIOC: thick, RLS dotted)