Analysis Models for Blind Search in Unstructured Overlays

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Abstract

Flooding and random walk are two basic mechanisms for blind search in unstructured peer-to-peer overlays. Although these mechanisms have been widely studied experimentally and via simulations, they have not been analytically modeled. Time overhead, message overhead, and success rate are often used as metrics for search schemes. This paper shows that node coverage is an important metric to estimate performance metrics such as the message efficiency, success rate, and object recall of a blind search. The paper then presents two simple models to analyze node coverage in random graph overlays. These models are useful to set query parameters, evaluate search efficiency, and to estimate object replication on a statistical basis.

1 Introduction

Peer-to-peer networks aim to allow Internet users to loosely organize themselves to share their resources with ease of implementation and maintenance [6]. Unstructured peer-to-peer overlays have the following advantages over structured overlays [9]: they can handle high churn rates easier, they do not incur much overhead for maintaining the logical structure, and they support keyword searches [8],[5] based on semantic identification and information retrieval techniques [10], and complex queries such as range queries. More realistic P2P applications have recently been developed with unstructured overlays than with structured ones due to these advantages. Flooding, random walk, and expanding ring, in conjunction with the Time-to-Live (TTL) constraint ([7]), are the widely used algorithms for *blind* search, (a.k.a. *unguided* search).

Existing studies of blind search are based on empirical rationale and the performance of blind search is studied primarily through simulations. Using extensive modeling and simulations, Lv et al. [7] studied the performance of P2P systems under three considerations: network topology, query distribution, and replication distribution. A study of random walk based on graph theory and Chernoff bounds compared the performance of random walk with that of flooding [4]. The study shows that the effect of a k-step random walk is statistically similar to that of taking k independent samples in a well-connected graph. Using this approximation of independent sampling, simplified formulae for the *success rate*, *message overhead*, and *time overhead* of random walk as functions of TTL, object popularity, and number of walkers were given [2]. The *success rate* was statistically characterized by the probability of a successful search, $p_s = 1 - (1-c)^{kT}$, where c is the popularity of the object (i.e., the ratio of the nodes that have an object copy), T is the TTL, and k is the number of random walkers. This approach relies on an accurate estimate of the popularity, which is usually not available.

Contributions. This paper shows that *node coverage* is a useful metric in analyzing the performance of blind searches – specifically, to estimate the message efficiency, success rate, and object recall. *Node coverage* is defined as the fraction of nodes visited in a query search. There has been no analytic model for node coverage computation. The paper then formulates two simple theoretical models for analyzing the properties of the random walk and flooding search methods in unstructured P2P networks. These models are based on node coverage analysis on the *random graph* topology, which is a small-world network. The models provide a guideline to understand how the settings of the querying parameters and network characteristics impact the efficiency of the search strategies. This will allow system designers to tune parameters to achieve desired performance trade-offs and to estimate object replication on a statistical basis.

Random graph topology. We assume a random graph overlay [3], as this is the simplest unstructured topology underlying all small-world models, and can offer reasonable guidelines for more refined topologies. The degree distribution among nodes in a random graph is much more uniform than that in a Power-Law random graph and a Gnutella Graph [7]. In the simplest random graph model, there exists a link with probability p between any two nodes. For a graph that has N nodes, the expected number of links is: $p\frac{N(N-1)}{2}$ and the average node degree d is: $p(N-1)$.

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N: total number of nodes p: probability of a link between two nodes (random graph) d: average degree of a node w: number of random walkers r: number of object replicas x: number of query messages h: number of query hops v: number of visits to nodes	
$u, u(x), u(h), u(h,w)$: number of distinct nodes visited u/N: node coverage $p_s(h,w,r)$: success probability for a search	

Table 1. Notations for the models.

2 Node coverage

In a random graph overlay where object distributions are also random, the more nodes that a query process covers, the higher the chance that a specified object is found, and the more the expected number of copies of qualifying objects that can be retrieved. We define *node coverage* as the fraction of nodes visited by query messages. We express it as a function of parameters such as the number of query hops or the message overhead. Node coverage is independent of object characteristics and depends only on the search strategy and graph topology. We identify the following uses of node coverage.

1. Node coverage gives a more reasonable basis to calculate *success rate*, defined as the probability that a search yields a satisfactory object within the specified constraints, such as the number of hops or the message overhead. For example, "what is the probability that a search could find a matching object within 3 hops of message forwarding?" Knowing such answers is useful in setting the parameters such as TTL or the number of walkers for the search. Observe that a query may be forwarded to a node that it has already visited; thus the node coverage, rather than the message overhead, should serve as the basis for calculating the success rate (assuming the object exists in the network).

2. For keyword searches and range searches, it is often desirable to find as many objects as possible (the *recall* metric) that satisfy the search criteria. *Message efficiency*, defined as the number of qualifying objects retrieved (i.e., recall) per query message, is another important metric in such environments. The recall is estimated by using the node coverage and the total number of qualifying objects in the network.

3. It may happen that a queried object does not exist in the network. A high node coverage without a query success indicates a high likelihood of this condition. This can be used as a guideline to call off the search.

4. The statistics of node coverage of a search along with that of recall can be used to estimate the replication ratio of an object. Low node coverage and high recall implies a high replication ratio and vice-versa.

The notation used in this paper is given in Table 1.

3 The algebraic model

This model performs a node coverage analysis but makes no distinction among the search methods when node coverage is computed. Each query message is treated as an independent sample. This model gives the expected node coverage in terms of the message overhead, and then in terms of the hop count.

The first hop of message forwarding always covers $w+1$ distinct nodes in random walk, or $d+1$ expected nodes in flooding (including the initiator). Due to the randomness of node links, from the second hop onwards, a message forwarding may visit a node that has already been visited.

Suppose a randomly chosen link is being probed by a message, and u is the expected number of distinct nodes that have been visited so far. The probability that a new node is discovered by this message is $\frac{N-u}{N-2}$. Thus, the expected number of distinct nodes visited would be $u + \frac{N-u}{N-2}$ after this message.

Let x denote the number of query messages so far. Then:

$$u(x+1) = u(x) + \frac{N - u(x)}{N-2} 1 \quad (1)$$

which can be approximated as:

$$u'(x) = \frac{N}{N-2} - \frac{u(x)}{N-2} \quad (2)$$

This equation can be solved as:

$$u(x) = Ce^{-\frac{x}{N-2}} + N \quad (3)$$

Here C is a constant determined by the initial condition.

Random Walkers. Within the first hop, $w+1$ distinct nodes (including the initiator itself) are visited. The initial condition takes the following form:

$$u(w+1) = w+1 \quad (4)$$

We can then solve for the constant C:

$$C = (w+1-N)e^{\frac{w+1}{N-2}} \quad (5)$$

Then the complete solution for equation (3) is:

$$u(x) = \begin{cases} N - (N-w-1)e^{\frac{w+1-x}{N-2}} & \text{if } x > w+1 \\ x & \text{if } x \le w+1 \end{cases} \quad (6)$$

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If we express u in terms of h, where h is the number of query hops and $x = wh + 1$, we obtain:

$$u(h) = \begin{cases} N - (N - w - 1)e^{\frac{w(1-h)}{N-2}} & \text{if } h > 1 \\ w + 1 & \text{if } h = 1 \end{cases} \quad (7)$$

Flooding. The difference between $u(x)$ for random walk and for flooding is that for flooding, message overhead is exponential in number of hops.

The initial condition for Equation (3) for flooding is:

$$u(d + 1) = d + 1 \quad (8)$$

We can then solve for the constant C:

$$C = (d + 1 - N)e^{\frac{d+1}{N-2}} \quad (9)$$

Then the complete solution for Equation (3) is:

$$u(x) = \begin{cases} N - (N - d - 1)e^{\frac{d+1-x}{N-2}} & \text{if } x > w + 1 \\ x & \text{if } x \leq w + 1 \end{cases} \quad (10)$$

As $x = \sum_{i=0}^{h} d^i = \frac{d^{h+1}-1}{d-1}$, we can express u in terms of h as:

$$u(h) = \begin{cases} N - (N - d - 1)e^{\frac{d^2-d^{h+1}}{(d-1)(N-2)}} & \text{if } h > 1 \\ d + 1 & \text{if } h = 1 \end{cases} \quad (11)$$

Replication. We assume random replication – r replicas of an object are randomly distributed in the network. The probability of finding a replica then becomes:

$$p_s(h) = 1 - (1 - \frac{u(h)}{N})^r \quad (12)$$

The term $u(h)/N$ is the node coverage.

4 Combinatorial model for random walk

In an unstructured overlay with random graph topology, the expected success rate of a specific query depends on the fraction of nodes covered by the search and the number of copies of the queried object. In this model, we derive the expected *node coverage* in terms of the number of message hops. The coverage analysis begins by analyzing the behavior of a single random walker and then extends the results to multiple walkers. The behavior of a single walker can be treated as a random sampling and multiple walkers are considered to be independent. For a single random walker, if we consider the node coverage as a state variable, the state after the next hop depends only on the current state. The walk can thus be modeled as a Markov process.

Let v be the number of nodes visited so far. Let $Pr(u, v)$ denote the probability that after v node visits, u distinct nodes have been visited.

For the first hop ($v = 2$):

- $Pr(2, 2) = 1$.

For the second hop ($v = 3$):

- $Pr(2, 3) = Pr(2, 2) \cdot 0 = 0$
- $Pr(3, 3) = Pr(2, 2) \cdot 1 = 1$

For the third hop ($v = 4$):

- $Pr(3, 4) = Pr(3, 3) \cdot \frac{3-2}{N-2}$
- $Pr(4, 4) = Pr(3, 3) \cdot \frac{N-3}{N-2}$

For the fourth hop ($v = 5$):

- $Pr(3, 5) = Pr(3, 4) \cdot \frac{3-2}{N-2}$
- $Pr(4, 5) = Pr(4, 4) \cdot \frac{4-2}{N-2} + Pr(3, 4) \cdot \frac{N-3}{N-2}$
- $Pr(5, 5) = Pr(4, 4) \cdot \frac{N-4}{N-2}$

For the fifth hop ($v = 6$):

- $Pr(3, 6) = Pr(3, 5) \cdot \frac{3-2}{N-2}$
- $Pr(4, 6) = Pr(4, 5) \cdot \frac{4-2}{N-2} + Pr(3, 5) \cdot \frac{N-3}{N-2}$
- $Pr(5, 6) = Pr(5, 5) \cdot \frac{5-2}{N-2} + Pr(4, 5) \cdot \frac{N-4}{N-2}$
- $Pr(6, 6) = Pr(5, 5) \cdot \frac{N-5}{N-2}$

Based on this pattern, the inductive expression for probability $Pr(u, v)$ is given in Figure 1.

Define the expected number of distinct nodes covered by w random walkers after traveling h hops to be $\overline{u}(h, w)$. Then $\overline{u}(h, 1)$ can be expressed as follows. (Note that $h = v - 1$.)

$$\overline{u}(h, 1) = \begin{cases} h + 1 & \text{if } h \leq 2 \\ \sum_{i=3}^{h+1} Pr(i, h + 1) \cdot i & \text{if } h > 2 \end{cases} \quad (14)$$

Assuming r replicas of the desired object, the success rate for a single walker is expected to be:

$$p_s(h, 1, r) = 1 - \left[1 - \frac{\overline{u}(h, 1)}{N}\right]^r \quad (15)$$

Consider $w > 1$ random walkers. When all walkers are modeled to walk synchronously, the node coverage state variable of the Markov process gets a complex distribution and incurs high calculation overhead. We adopt a simpler approach.

As multiple ($w > 1$) walkers travel independent of each other, any walker is expected to visit $\overline{u}(h, 1)$ "distinct" nodes after h hops of forwarding. Here, "distinct" refers to the nodes witnessed by a single walker. It is possible that some of those $\overline{u}(h, 1)$ nodes have been visited by other walkers. To compute $\overline{u}(h, w)$ for $w > 1$, assume they travel the network sequentially.

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$$Pr(u,v) \text{ (for } u \leq v) = \begin{cases} 1 & \text{if } u=2, v=2 \\ Pr(u,v-1)\cdot\frac{u-2}{N-2} & \text{if } u=2, v\neq 2 \\ Pr(u-1,v-1)\cdot\frac{N-(u-1)}{N-2} + Pr(u,v-1)\cdot\frac{u-2}{N-2} & \text{if } 2<u<v \\ Pr(u-1,v-1)\cdot\frac{N-(u-1)}{N-2} & \text{if } u=v>2 \end{cases} \quad (13)$$

Figure 1. $Pr(u,v)$ for the combinatorial model.

- After the first walker finishes, $\overline{u}(h,1)$ distinct nodes were visited. Let $new(h,1) = \overline{u}(h,1)$.
- The second walker sees $\overline{u}(h,1)$ distinct nodes according to its own witness. Among those $\overline{u}(h,1)$ nodes, $(1-\frac{\overline{u}(h,1)}{N})\overline{u}(h,1)$ are expected to be new from the previous walk. Denote these by $new(h,2)$.
- For the i^{th} walker, the expected number of additional distinct nodes it visits is:

$$new(h,i) = \left[1 - \frac{\sum_{k=1}^{i-1} new(h,k)}{N}\right]\overline{u}(h,1) \quad (16)$$

- The expected total number of distinct nodes visited by w random walkers is:

$$\overline{u}(h,w) = \sum_{i=1}^{w} new(h,i) \quad (17)$$

- The success rate (defined as the probability that at least one of the r copies of the desired object is found by the w walkers), can be expressed using node coverage:

$$p_s(h,w,r) = 1 - \left(1 - \frac{\overline{u}(h,w)}{N}\right)^r \quad (18)$$

This model has computational complexity $O(N^2)$ due to the nature of Equations (13) and (14). In contrast, the algebraic model has complexity $O(1)$.

5 Conclusions

The contributions were summarized in Section 1. Some of the following ongoing work is reported in [11].

1. Compare the accuracy of the two proposed models with each other, and with experiments/simulations.
2. Compare the accuracy of success probability as computed using our formulations of node coverage, with the earlier formulation of [2] that used popularity estimates and assumed independent sampling.
3. Analyze other overlay topologies, such as scale-free networks and other small-world networks [1]. These have power-law distribution for node degree, small average path lengths, and high clustering coefficients.
4. Explore analytical expressions for other metrics that can be derived from node coverage.

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