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Research Note

Reasoning about causality between distributed nonatomic events

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Abstract

The complex events in distributed applications such as industrial process control, avionics, navigation, planning, robotics, diagnostics, virtual reality, and temporal and geographic databases, are realistically modeled by nonatomic events. This paper derives and studies causality relations between nonatomic distributed events in the execution of a complex distributed application. Such causality relations are useful because they provide a fine level of discrimination in the specification of the relative timing relations and synchronization conditions between the nonatomic events. The paper then proposes a set of axioms on the proposed causality relations. The set of axioms provides a mechanism for temporal and spatial reasoning with the set of relations and can be used to derive all possible implications from any valid predicate on the proposed relations. © 1997 Elsevier Science B.V.

Keywords: Time; Causality; Distributed system; Temporal and spatial reasoning; Interval

1. Introduction

The notion of grouping elementary events in a system execution into higher level nonatomic events is useful for event abstraction. Nonatomic events are modeled in distributed applications such as industrial process control, navigation, planning, robotics, virtual reality, coordination in mobile systems, and temporal and geographic databases. These applications deal with nonatomic events that are nonlinear, i.e., nonatomic events for which at least some of their component atomic events occur at more than a single point in space concurrently [11, 14]. For these applications, the traditional causality relation [24] defined between individual points in space-time is inadequate when applied to

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the abstracted nonatomic events, for the following reasons. (i) The interaction between two nonatomic events cannot be captured at a fine level of discrimination using various degrees of relative timing constraints, as required for a sophisticated and realistic modeling of these applications. (ii) The synchronization conditions between two nonatomic events cannot be richly specified using various degrees of relative timing constraints, as required for an intelligent and realistic modeling of these applications. Therefore, a rich set of causality relations that allows the expression of various degrees of relative timing constraints to accurately represent and specify real-life relationships between nonatomic events needs to be defined. We also need a corresponding system of axioms to reason intelligently with the proposed relations. We propose and examine causality relations between nonatomic nonlinear events in a distributed system and provide a set of axioms to reason with such relations.

We use the space-time model for a system execution. This model is a poset event structure model as in [4, 11, 14, 26]. Consider a poset $(E, <)$ where $<$ is an irreflexive partial ordering that can be either the precedence relation "precedes", or the causality relation "causes" which is a modalized precedence relation ("precedes" plus some modal statement). We prefer to let $<$ denote the causality relation because the nonatomic events between which relations are derived are application-specific groupings of atomic events, and an application-specific modal statement is associated with the precedence relation. Thus, $(E, <)$ represents points in space-time which are the most primitive atomic events related by the causality relation. Elements of E are partitioned into local executions at coordinates in the space dimensions. Each local execution E_i is a linearly ordered set of events in partition i. An event e in partition i is denoted e_i.

Nonatomic nonlinear events are now defined in our system model as follows. Let $\mathcal{E}$ denote the power set of E. Let $\mathcal{A}(\neq \emptyset) \subseteq (\mathcal{E} - \emptyset)$. Thus, there is an implicit one-many mapping from $\mathcal{A}$ to E. Each element A of $\mathcal{A}$ is a nonempty subset of E, and is termed an *interval* or a *nonatomic event.* [1]

Relations between time durations and between instants have been extensively studied in the literature on time and knowledge representation, temporal reasoning, and interval algebras; several axiom systems have been proposed for these relations. Some representative literature is [1-8, 12, 13, 15-18, 20-25]. This literature studied causality relations between instants or between durations at a single point in space, or between intervals in relativistic space-time. In all these studies, causality relations were considered only between linear intervals or durations.

The study of nonatomic poset intervals, i.e., nonatomic nonlinear intervals, is important in developing intelligence and in knowledge representation and reasoning in a distributed system for applications described earlier. Time in nonatomic poset intervals has been studied in the context of convex and nonconvex intervals in two or more dimensions [24]. Properties of time in nonlinear intervals have also been studied in the context of branching time which represents different possible courses of events with only one course happening [9, 24]. However, in the context of relativistic space-time which is our system model, there is no prior treatment of nonlinear time or causality relations between nonatomic poset intervals, except for the following.

[1] We will use the term "interval" interchangeably with "event" when referring to nonatomic events.

Table 1
Relations in [11]

Relation r	Expression for $r(X,Y)$
$R1$	$\forall x \in X\ \forall y \in Y,\ x < y$
$R1'$	$\forall y \in Y\ \forall x \in X,\ x < y$
$R2$	$\forall x \in X\ \exists y \in Y,\ x < y$
$R2'$	$\exists y \in Y\ \forall x \in X,\ x < y$
$R3$	$\exists x \in X\ \forall y \in Y,\ x < y$
$R3'$	$\forall y \in Y\ \exists x \in X,\ x < y$
$R4$	$\exists x \in X\ \exists y \in Y,\ x < y$
$R4'$	$\exists y \in Y\ \exists x \in X,\ x < y$

Table 2
Inclusion relationships between relations, from [11]

Relation of row header to column header	$R1$	$R2$	$R3$	$R4$
$R1$	$=$	$\sqsubseteq$	$\sqsubseteq$	$\sqsubseteq$
$R2$	$\sqsupseteq$	$=$	$\parallel$	$\sqsubseteq$
$R3$	$\sqsupseteq$	$\parallel$	$=$	$\sqsubseteq$
$R4$	$\sqsupseteq$	$\sqsupseteq$	$\sqsupseteq$	$=$

Lamport defined system executions using two relations $\longrightarrow$ and $\dashrightarrow$ between primitive nonatomic elements and provided axioms on these relations [14]. Informally, these relations are as follows. Let a nonatomic event be a set of atomic events. For two nonatomic events X and Y in $\mathcal{A}$, $X \longrightarrow Y$ iff every atomic event in X causally precedes every atomic event in Y. $X \dashrightarrow Y$ iff some atomic event in X causally precedes some atomic event in Y. The axioms in [14] were examined in [1].

In an earlier paper [11], we showed that the two relations defined by Lamport are not sufficient to capture the essential temporal properties of system executions and specify relative timing constraints between nonatomic events in distributed systems. In [11], we proposed a set of new relations between nonatomic events in a distributed system to capture a spectrum of relative timing constraint specifications, without assuming a global time axis. These relations $R1$–$R4$ and $R1'$–$R4'$ from [11] are expressed in terms of the quantifiers over X and Y in Table 1.

Note that all the relations in Table 1 are not independent relations. Table 2 gives the inclusion relationship among the causality relations $R1$–$R4$, where each cell in the grid indicates the relationship of the row header to the column header. The notation for the inclusion relationship between causality relations on nonatomic events is as follows. The inclusion relation "is a subrelation of" is denoted "$\sqsubseteq$". "$\sqsupseteq$" is the inverse of $\sqsubseteq$. "=" stands for equality between relations in addition to its standard usage as the equality in other contexts. For two causality relations r_1 and r_2, we define $r_1 \parallel r_2$ to be $(r_1 \not\sqsubseteq r_2 \wedge r_2 \not\sqsubseteq r_1)$. The relations $\{R1, R2, R3, R4\}$ form a lattice hierarchy ordered by $\sqsubseteq$.

Table 1 also defined relations $R1'$, $R2'$, $R3'$, and $R4'$, for which the order of quantifiers was reversed from the order in $R1$, $R2$, $R3$, and $R4$, respectively. Observe that the relations $R2'$ and $R3'$ are different from relations $R2$ and $R3$, respectively, when applied to posets. However, for a linear interval, they are the same as $R2$ and $R3$, respectively. $R1'$ and $R4'$ are the same as $R1$ and $R4$, respectively.

In this paper, we extend the above hierarchy of relations in [11] by presenting new relations that form an "exhaustive" set of causality relations between nonatomic poset events using first-order predicate logic (Section 2). We derive a complete axiom system to reason with these relations (Section 3). Section 4 concludes. The results of this paper are included in [10].

2. Relations between nonatomic poset events

Let $\mathcal{A}$ be the set of all the sets that represent a higher level grouping of the events of E that is of interest to the particular application. An element of $\mathcal{A}$ is denoted A.

Definition 1. An interval A is linear iff $\forall x, y \in A, x \leqslant y \vee y \leqslant x$.

Definition 2. An interval A is convex iff $\forall x, y \in A, \forall z \in E\ (x < z < y) \Longrightarrow z \in A$.

Definition 3. N_A, the node set of interval A, is $\{i \mid E_i \cap A \neq \emptyset\}$.

Our results apply to nonlinear, i.e., poset, intervals. These intervals need not be convex because convexity is not important in the study of causality. The cardinality of the node set of the intervals we consider is greater than one.

The relations in [11] are used to derive an exhaustive set of causality relations between nonatomic poset events, denoted $\mathcal{R}$. As an intermediate step, we propose definitions of certain proxies of a nonatomic event in Section 2.1.

2.1. Proxies of nonatomic poset events

In the extensive literature on linear intervals and time durations, for example [2–8, 12, 13, 15–18, 20–25], a convex linear interval is identified by the instants of its beginning and end, whereas a nonconvex linear interval is identified by the beginning and end instants of its convex subsets. The beginning and end instants of a linear interval are points in space-time which are atomic events in E. For a nonatomic poset interval, it is natural to identify counterparts for the beginning and end instants. These counterparts will serve as "proxy" events for the poset interval just as the events at the beginning and end of linear intervals such as time durations serve as proxies for the linear interval. The proxies identify the durations on each node, in which the nonatomic event occurs.

We now define two proxies corresponding to the beginning and end of a nonatomic poset interval [10].

Definition 4.

$$L_X = \{e_i \in X \mid \forall e_i' \in X, e_i \leqslant e_i'\},$$

$$U_X = \{e_i \in X \mid \forall e_i' \in X, e_i \geqslant e_i'\}.$$

For any poset X, U_X and L_X are the sets of the maximal elements in X for each node and the set of the minimal elements in X for each node, respectively. U_X and L_X correspond to the end of the poset and the beginning of the poset, respectively, and can act as a *proxy* for poset X, depending on context and application. As per Definition 4, each of L_X and U_X contains one event from each node in N_X.

An equally valid interpretation of the beginning and end of a poset are the sets of its minimal and maximal elements, respectively, as defined by the irreflexive partial order across the nodes. This leads to the following alternate definition of the proxies L_X and U_X.

Definition 5.

$$L_X = \{e \in X \mid \forall e' \in X, e \not\succ e'\},$$

$$U_X = \{e \in X \mid \forall e' \in X, e \not\prec e'\}.$$

L_X is the largest anti-chain containing the minimal elements of X. U_X is the largest anti-chain containing the maximal elements of X.

The causality relations between poset intervals will be derived using proxies and will depend on whether proxies are defined by Definition 4 or by Definition 5. The distinction between the two resulting sets of relations is studied in Section 3.1. However, in all results and discussions upto Section 3.1, we assume that any one of these definitions of proxies is consistently used.

2.2. *Deriving the relations*

We propose that there are two aspects of a relation that can be specified between poset intervals. One aspect deals with the determination of an appropriate *proxy* for each interval. A good choice for the proxy(ies) of the interval are the beginning and end of the interval (Definition 4 or 5), as justified in Section 2.1. Relations between posets are not specified on members $Z \in \mathcal{A}$, but rather on their proxies U_Z and L_Z. The second aspect of specifying relations $r(X,Y)$ specifies how the chosen proxies of X and Y are related. Fig. 1 depicts the proxies of X and Y and serves as a visual aid for the following discussion; recall that each poset X and Y represents a grouping of atomic events of interest to the application. We will use the following notation in the ensuing formulation of the relations between poset intervals. C_k^n is the number of combinations of k things out of n things. P_k^n is the number of permutations of k things out of n things.

A proxy for X and Y can be chosen in $C_1^2 \times C_1^2$ ways; it can be the set of maximal elements or minimal elements for each of X and Y. This is the first aspect of specifying relations between posets, and corresponds to the relations in $\{R1, R2, R3, R4\}$. From Table 2, these four relations form a lattice ordered by $\sqsubseteq$.

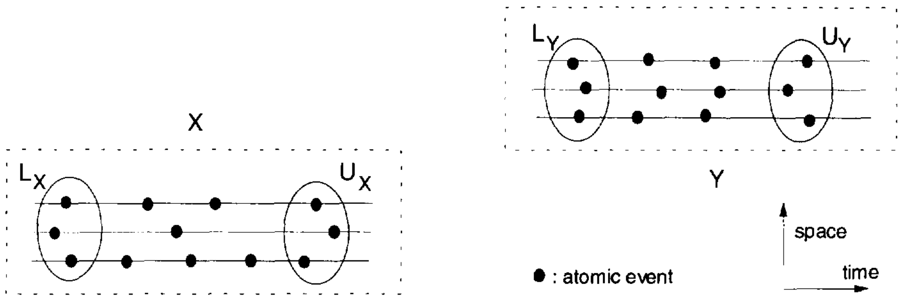

Fig. 1. Poset events X and Y and their proxies.

Table 3
Full hierarchy of relations of Table 1 [11]; relations $R1$, $R1'$, $R2$, $R2'$, $R3$, $R3'$, $R4$, $R4'$ of Table 1 are renamed a, a', b, b', c, c', d, d', respectively; relations in the row and column headers are defined between X and Y

Relation names: its quantifiers for $x < y$	$R1, a$ $(= R1', a')$: $\forall x \forall y$ $(= \forall y \forall x)$	$R2, b$: $\forall x \exists y$	$R2', b'$: $\exists y \forall x$	$R3, c$: $\exists x \forall y$	$R3', c'$: $\forall y \exists x$	$R4, d$ $(= R4', d')$: $\exists x \exists y$ $(= \exists y \exists x)$
$R1, a$ $(= R1', a')$: $\forall x \forall y$ $(= \forall y \forall x)$	$=$	$\sqsubseteq$	$\sqsubseteq$	$\sqsubseteq$	$\sqsubseteq$	$\sqsubseteq$
$R2, b$: $\forall x \exists y$	$\sqsupseteq$	$=$	$\sqsupseteq$	$\parallel$	$\parallel$	$\sqsubseteq$
$R2', b'$: $\exists y \forall x$	$\sqsupseteq$	$\sqsubseteq$	$=$	$\parallel$	$\parallel$	$\sqsubseteq$
$R3, c$: $\exists x \forall y$	$\sqsupseteq$	$\parallel$	$\parallel$	$=$	$\sqsubseteq$	$\sqsubseteq$
$R3', c'$: $\forall y \exists x$	$\sqsupseteq$	$\parallel$	$\parallel$	$\sqsupseteq$	$=$	$\sqsubseteq$
$R4, d$ $(= R4', d')$: $\exists x \exists y$ $(= \exists y \exists x)$	$\sqsupseteq$	$\sqsupseteq$	$\sqsupseteq$	$\sqsupseteq$	$\sqsupseteq$	$=$

The second aspect of specifying the causality relations between posets deals with how the atomic elements of the chosen proxies of X and Y are related by causality. There are $C_1^2 \times C_1^2$ combinations of distinct quantifications $\exists$ and $\forall$ over the proxies of X and Y to express $r(X,Y)$, and for each combination, there are P_1^2 permutations of the proxies of X and Y. The eight relations so formed are exactly the relations $R1$, $R1'$, $R2$, $R2'$, $R3$, $R3'$, $R4$, $R4'$ of Table 1 and are renamed a, a', b, b', c, c', d, d', respectively, to avoid confusion with their original names used for the first aspect of specifying the relations between poset intervals. Table 3 gives the hierarchy and inclusion relationship among the relations in $\{a, a', b, b', c, c', d, d'\}$. Each cell in the table indicates the relation of the row header to the column header. The notation used is the same as that used for Table 2. Note that a' and d' are the same as a and d, respectively. Observe that the six distinct relations form a lattice ordered by $\sqsubseteq$.

The proposed relations between nonatomic poset events are given in the second column of Table 4. The relations are formed by combining the two aspects of deriving causality relations as described above, and are labeled in the first column of Table 4 as

Table 4
Proposed relations $r(X,Y)$ in $\mathcal{R}$

Relation $r(X,Y)$	Relation definition specified by quantifiers for $x < y$, where $x \in X$, $y \in Y$
$R1a$	$\forall x \in U_X \ \forall y \in L_Y$
$R1a'$ (= $R1a$)	$\forall y \in L_Y \ \forall x \in U_X$
$R1b$	$\forall x \in U_X \ \exists y \in L_Y$
$R1b'$	$\exists y \in L_Y \ \forall x \in U_X$
$R1c$	$\exists x \in U_X \ \forall y \in L_Y$
$R1c'$	$\forall y \in L_Y \ \exists x \in U_X$
$R1d$	$\exists x \in U_X \ \exists y \in L_Y$
$R1d'$ (= $R1d$)	$\exists y \in L_Y \ \exists x \in U_X$
$R2a$	$\forall x \in U_X \ \forall y \in U_Y$
$R2a'$ (= $R2a$)	$\forall y \in U_Y \ \forall x \in U_X$
$R2b$	$\forall x \in U_X \ \exists y \in U_Y$
$R2b'$	$\exists y \in U_Y \ \forall x \in U_X$
$R2c$	$\exists x \in U_X \ \forall y \in U_Y$
$R2c'$	$\forall y \in U_Y \ \exists x \in U_X$
$R2d$	$\exists x \in U_X \ \exists y \in U_Y$
$R2d'$ (= $R2d$)	$\exists y \in U_Y \ \exists x \in U_X$
$R3a$	$\forall x \in L_X \ \forall y \in L_Y$
$R3a'$ (= $R3a$)	$\forall y \in L_Y \ \forall x \in L_X$
$R3b$	$\forall x \in L_X \ \exists y \in L_Y$
$R3b'$	$\exists y \in L_Y \ \forall x \in L_X$
$R3c$	$\exists x \in L_X \ \forall y \in L_Y$
$R3c'$	$\forall y \in L_Y \ \exists x \in L_X$
$R3d$	$\exists x \in L_X \ \exists y \in L_Y$
$R3d'$ (= $R3d$)	$\exists y \in L_Y \ \exists x \in L_X$
$R4a$	$\forall x \in L_X \ \forall y \in U_Y$
$R4a'$ (= $R4a$)	$\forall y \in U_Y \ \forall x \in L_X$
$R4b$	$\forall x \in L_X \ \exists y \in U_Y$
$R4b'$	$\exists y \in U_Y \ \forall x \in L_X$
$R4c$	$\exists x \in L_X \ \forall y \in U_Y$
$R4c'$	$\forall y \in U_Y \ \exists x \in L_X$
$R4d$	$\exists x \in L_X \ \exists y \in U_Y$
$R4d'$ (= $R4d$)	$\exists y \in U_Y \ \exists x \in L_X$

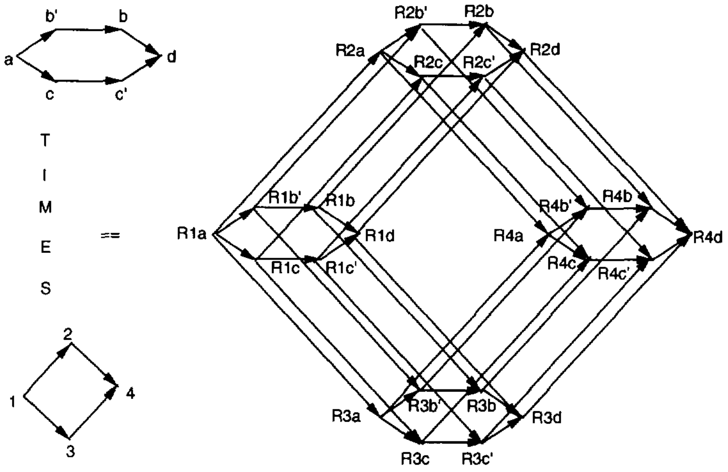

Fig. 2. Hierarchy of proposed relations.

follows. The relations $R1$, $R2$, $R3$, and $R4$ on linear intervals correspond to the groups of relations $R1*$, $R2*$, $R3*$ and $R4*$, respectively, for poset intervals. The hierarchy among the relations $R1*$, $R2*$, $R3*$ and $R4*$ is isomorphic to the hierarchy among $R1$, $R2$, $R3$, and $R4$.

Relations $R1*(X,Y)$ relate certain activity of U_X to certain activity of L_Y. Specifically, $R1*(X,Y)$ could be specified by quantifying over all or some elements of U_X, all or some elements of L_Y, and the order of the quantifications of the proxies of X and Y can be permuted. There are eight possibilities for $R1*(X,Y)$, that correspond to relations $\{a,a',b,b',c,c',d,d'\}$. $R2*(X,Y)$ relate certain activity of U_X to certain activity of U_Y. $R3*(X,Y)$ relate certain activity of L_X to certain activity of L_Y. $R4*(X,Y)$ relate certain activity of L_X to certain activity of U_Y. For each of $R2*(X,Y)$, $R3*(X,Y)$, and $R4*(X,Y)$, there are eight possible relations like for $R1*(X,Y)$.

The relations $\{R1*,R2*,R3*,R4*\}$ between proxies for X and Y, and the relations $\{a,a',b,b',c,c',d,d'\}$ between the elements of the proxies, when multiplied give 32 relations over the domain $\mathcal{A}\times\mathcal{A}$ to express $r(X,Y)$. The resulting set of poset relations, denoted $\mathcal{R}$ and given in the second column of Table 4, is thus a product of the relations represented by the two lattices $\{R1*,R2*,R3*,R4*\}$ and $\{a,b,b',c,c',d\}$ of unique elements, as shown in Fig. 2. The resulting hierarchy of 24 unique relations forms a lattice $(\mathcal{R},\sqsubseteq)$ and provides a fine-grained choice of causality relations for specification of relative timing and synchronization conditions.

2.3. Discussion

The set of relations we formulated between nonatomic poset events is exhaustive using first-order predicate logic. The proposed relations form a lattice hierarchy. The strongest relation is $R1a$ and the weakest is $R4d$. The significance of a relation $R?\#\ (X,Y)$ is determined by examining ? for the choice of proxies of X and Y, and examining # for how these proxies are related. The proposed set of causality relations between nonatomic

Table 5
Reflexivity, symmetry and transitivity of $R1$, $R2$, $R3$, $R4$ from [11]

Relation	Reflexive?	Symmetric?	Transitive?
R1 [14]	no	no	yes
R2	no	no	yes
R3	no	no	yes
R4 [14]	no	no	no

poset events is richer than the specific causality relations in the literature. The suite of two relations in [14], viz., $\longrightarrow$ and $--\rightarrow$, correspond to $R1a$ and $R4d$, respectively. The suite of relations in [11] and listed in Table 1 correspond to the new relations as follows: $R1 = R1'$, $R2$, $R2'$, $R3$, $R3'$, $R4 = R4'$ correspond to $R1a$, $R2b$, $R2b'$, $R3c$, $R3c'$, $R4d$, respectively. The significance of the complete hierarchy of causality relations in first-order predicate logic is given in Section 4.

3. Axiom system

The inclusion hierarchy of the relations in Table 4 is pictorially depicted in Fig. 2. This hierarchy is captured by the following axioms XP1–XP6. Let V_1 denote the set $\{1,2,3,4\}$ and let V_2 denote the set $\{a,b,b',c,c',d\}$. Then the axioms are:

XP1. $R1? \sqsubseteq R2? \sqsubseteq R4?$, where ? is instantiated from V_2.

XP2. $R1? \sqsubseteq R3? \sqsubseteq R4?$, where ? is instantiated from V_2.

XP3. $R2? \parallel R3\#$, where ? and # are separately instantiated from V_2.

XP4. $R?a \sqsubseteq R?b' \sqsubseteq R?b \sqsubseteq R?d$, where ? is instantiated from V_1.

XP5. $R?a \sqsubseteq R?c \sqsubseteq R?c' \sqsubseteq R?d$, where ? is instantiated from V_1.

XP6. $R?b \parallel R?c'$, $R?b' \parallel R?c'$, $R?b \parallel R?c$, $R?b' \parallel R?c$, where ? is instantiated from V_1.

Further axioms for the relations in Table 4 are derived from Tables 5, 6, and 7 as follows. Table 5 is reproduced from [11] and represents the reflexivity, symmetry, and transitivity for the relations $R1$–$R4$ defined in [11]. Table 6 is reproduced from [11] and gives the transitive axioms on the relations $R1$–$R4$ defined in [11]. Table 7 indicates that if the proxies of X and Y in $r_1(X,Y)$ are related by the row header of the table, and if the proxies of Y and Z in $r_2(Y,Z)$ are related by the column header of the table, then the corresponding proxies of X and Z are related by the corresponding table entry; this entry is useful in deducing $r(X,Z)$. If $r_1(X,Y)$ and $r_2(Y,Z)$, then the transitive relation $r(X,Z)$ is determined by the algorithm *Trans_Poset_Axioms* using Tables 5, 6, and 7 as follows.

Table 6
Axioms for causality relations $R1$, $R2$, $R3$, $R4$ from [11]

Axiom label	$r_1(X,Y) \wedge r_2(Y,Z) \Longrightarrow r(X,Z)$
AL1	$R1(X,Y) \wedge R2(Y,Z) \Longrightarrow R2(X,Z)$
AL2	$R1(X,Y) \wedge R3(Y,Z) \Longrightarrow R1(X,Z)$
AL3	$R1(X,Y) \wedge R4(Y,Z) \Longrightarrow R2(X,Z)$
AL4	$R2(X,Y) \wedge R1(Y,Z) \Longrightarrow R1(X,Z)$
AL5	$R3(X,Y) \wedge R1(Y,Z) \Longrightarrow R3(X,Z)$
AL6	$R4(X,Y) \wedge R1(Y,Z) \Longrightarrow R3(X,Z)$
AL7	$R2(X,Y) \wedge R3(Y,Z) \Longrightarrow true$
AL8	$R2(X,Y) \wedge R4(Y,Z) \Longrightarrow true$
AL9	$R3(X,Y) \wedge R2(Y,Z) \Longrightarrow R4(X,Z)$
AL10	$R4(X,Y) \wedge R2(Y,Z) \Longrightarrow R4(X,Z)$
AL11	$R3(X,Y) \wedge R4(Y,Z) \Longrightarrow R4(X,Z)$
AL12	$R4(X,Y) \wedge R3(Y,Z) \Longrightarrow true$

Algorithm Trans_Poset_Axioms.

1. Use the first two characters (*prefix*) of the identifier strings of $r_1(X,Y)$ and $r_2(Y,Z)$ as the inputs to Table 5 or 6.
   *temp*1 := output of the appropriate table.
   /* *temp*1 gives the relation between X and Z if X, Y, Z were all linear intervals.*/
   If *temp*1 = *true*, then $r(X,Z)$:= *true*; exit.
   /* no relation between X and Z can be inferred.*/
2. The row and column headers in Table 7 are the strings following the first two characters (*suffix*) of the identifier strings of the poset relations $\mathcal{R}$. Use the *suffixes* of $r_1(X,Y)$ and $r_2(Y,Z)$ as the row header and column header inputs, respectively, to Table 7.
   *temp*2 := the entry identified by the row and column headers.
   If *temp*2 = *true*, then $r(X,Z)$:= *true*; exit.
   /* no relation between X and Z can be inferred.*/
3. Concatenate the values of *temp*1 and *temp*2 to get the value of $r(X,Z)$.

Example 6. If $R1c'(X,Y) \wedge R3b(Y,Z)$ then the algorithm yields $R1d(X,Z)$. In step 1, the inputs to Table 6 are $R1$ and $R3$, and the output *temp*1 is $R1$. In step 2, the inputs to Table 7 are c' and b, and its output *temp*2 is d. Step 3 concatenates *temp*1 and *temp*2 to yield $R1d$.

Example 7. If $R2a(X,Y) \wedge R1d(Y,Z)$ then the algorithm yields $R1b'(X,Z)$. In step 1, the inputs to Table 6 are $R2$ and $R1$, and the output *temp*1 is $R1$. In step 2, the inputs to Table 7 are a and d, and its output *temp*2 is b'. Step 3 concatenates *temp*1 and *temp*2 to yield $R1b'$.

Table 7
Intermediate table to derive further axioms for poset relations $\mathcal{R}$; the relation names in the row and column headers are the suffixes of the poset relations $\mathcal{R}$ defined between X and Y

Relation name: its quantifiers for $x < y$	a $(= a')$: $\forall x \forall y$ $(= \forall y \forall x)$	b: $\forall x \exists y$	b': $\exists y \forall x$	c: $\exists x \forall y$	c': $\forall y \exists x$	d $(= d')$: $\exists x \exists y$ $(= \exists y \exists x)$
a $(= a')$: $\forall x \forall y$ $(= \forall y \forall x)$	a $(\forall x \forall y)$	b' $(\exists y \forall x)$	b' $(\exists y \forall x)$	a $(\forall x \forall y)$	a $(\forall x \forall y)$	b' $(\exists y \forall x)$
b: $\forall x \exists y$	a $(\forall x \forall y)$	b $(\forall x \exists y)$	b' $(\exists y \forall x)$	*true*	*true*	*true*
b': $\exists y \forall x$	a $(\forall x \forall y)$	b' $(\exists y \forall x)$	b' $(\exists y \forall x)$	*true*	*true*	*true*
c: $\exists x \forall y$	c $(\exists x \forall y)$	d $(\exists x \exists y)$	d $(\exists x \exists y)$	c $(\exists x \forall y)$	c $(\exists x \forall y)$	d $(\exists x \exists y)$
c': $\forall y \exists x$	c $(\exists x \forall y)$	d $(\exists x \exists y)$	d $(\exists x \exists y)$	c $(\exists x \forall y)$	c' $(\forall y \exists x)$	d $(\exists x \exists y)$
d $(= d')$: $\exists x \exists y$ $(= \exists y \exists x)$	c $(\exists x \forall y)$	d $(\exists x \exists y)$	d $(\exists x \exists y)$	*true*	*true*	*true*

Example 8. If $R3a(X,Y) \wedge R2b(Y,Z)$ then the algorithm yields $R4b'(X,Z)$. In step 1, the inputs to Table 6 are $R3$ and $R2$, and the output *temp*1 is $R4$. In step 2, the inputs to Table 7 are a and b, and its output *temp*2 is b'. Step 3 concatenates *temp*1 and *temp*2 to yield $R4b'$.

Example 9. If $R3b(X,Y) \wedge R2c'(Y,Z)$ then the algorithm yields *true*. In step 1, the inputs to Table 6 are $R3$ and $R2$, and the output *temp*1 is $R4$. In step 2, the inputs to Table 7 are b and c', and its output *temp*2 is *true*. Hence, no relation between X and Z can be inferred.

We specify the following axioms XP7–XP14 of the form $r_1(X,Y) \Longrightarrow r_2(Y,X)$ for the nonatomic poset events. For each relation $r_1(X,Y)$, we determine the strongest relation(s) $r_2(Y,X)$ that can be stated between Y and X in the hierarchy depicted in Fig. 2 (axioms XP1–XP6). Thus, given a relation between X and Y, the axioms give all possible relations between Y and X. The notation $\overline{R}$ indicates that the relation R is false. These axioms can be verified to be meaningful by examining each axiom with the aid of Fig. 1 which shows X and Y in two-dimensional space-time.

XP7. $R1a(X,Y) \vee R1b(X,Y) \vee R1b'(X,Y) \vee R1c(X,Y) \vee R1c'(X,Y) \Longrightarrow \overline{R4d}(Y,X)$.

XP8. $R1d(X,Y) \Longrightarrow \overline{R4b}(Y,X) \wedge \overline{R4c'}(Y,X)$.

XP9. $R2a(X,Y) \vee R2b(X,Y) \vee R2b'(X,Y) \vee R2c(X,Y) \vee R2c'(X,Y) \Longrightarrow \overline{R2d}(Y,X)$.

XP10. $R2d(X,Y) \Longrightarrow \overline{R2b}(Y,X) \wedge \overline{R2c'}(Y,X)$.

XP11. $R3a(X,Y) \vee R3b(X,Y) \vee R3b'(X,Y) \vee R3c(X,Y) \vee R3c'(X,Y) \Longrightarrow \overline{R3d}(Y,X)$.

XP12. $R3d(X,Y) \Longrightarrow \overline{R3b}(Y,X) \wedge \overline{R3c'}(Y,X)$.

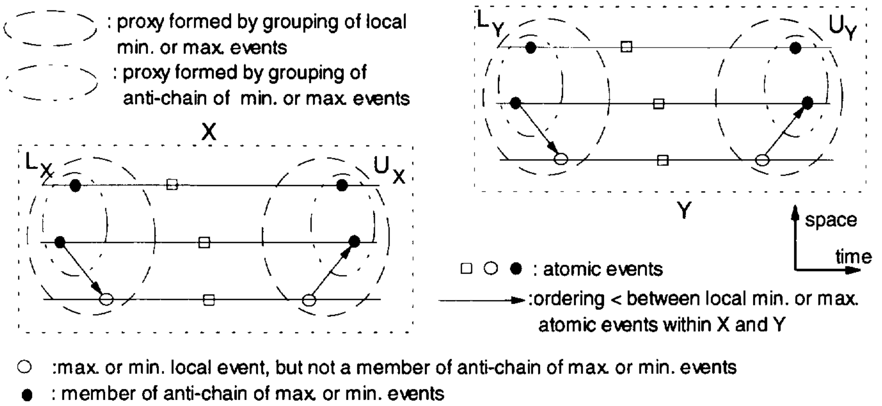

Fig. 3. Two definitions of proxies of poset events X and Y.

XP13. $R4a(X,Y) \vee R4b(X,Y) \vee R4b'(X,Y) \vee R4c(X,Y) \vee R4c'(X,Y) \Longrightarrow \overline{R1d}(Y,X)$.

XP14. $R4d(X,Y) \Longrightarrow \overline{R1b}(Y,X) \wedge \overline{R1c'}(Y,X)$.

In addition, we specify axiom XP15 that specifies the reflexivity and symmetry of the relations in $\mathcal{R}$.

XP15. The relations in $\mathcal{R}$ are not reflexive and are not symmetric.

$\mathcal{X}$ is the set of axioms XP1–XP6 (that specify hierarchy among relations), XP7–XP14 (that give all relations of the form $r_2(Y,X)$, given $r_1(X,Y)$), XP15 (that specifies reflexivity and symmetry), and the axioms that can be derived from algorithm *Trans_Poset_Axioms* to specify transitive relations. We do not attempt a completeness proof of this axiom system here. The axioms in $\mathcal{X}$ provide a "sufficiently" rich framework to reason about poset intervals because:

- Axioms XP1–XP6, XP7–XP14 and XP15 give all enumerations of relations $r(X,Y)$ as well as relations $r(Y,X)$, implied by $R(X,Y)$, $\forall r \forall R \in \mathcal{R}$.
- Algorithm *Trans_Poset_Axioms* enumerates all relations $r(X,Z)$ implied by $r_1(X,Y) \wedge r_2(Y,Z)$, $\forall r \forall r_1 \forall r_2 \in \mathcal{R}$.
- This set of axioms can be used to derive all possible implications from any given valid predicates on relations in $\mathcal{R}$.

3.1. Dependence on proxy definition

The derivation of the causality relations in $\mathcal{R}$ used the definition of proxies to represent the beginning and end of a nonatomic poset event. However, there are two equally meaningful definitions of proxies, as given by Definitions 4 and 5. Each application chooses the definition more suitable to it. Depending on which definition is used, we get two different sets of 32 relations $\mathcal{R}$, each of which satisfies the same hierarchy of Fig. 2 and the same set of axioms $\mathcal{X}$. The set $\mathcal{R}$ of 32 relations, each member relation in $\mathcal{R}$, and the axioms $\mathcal{X}$ on these relations, that are obtained by the use of Definition 4 (respectively, Definition 5) for proxies are identified by the superscript $\prec_i$ (respectively, $\prec$).

Next, we present axioms on the inclusion relationship $\sqsubseteq$ between relations in $\mathcal{R}^{\prec}$ and relations in $\mathcal{R}^{\prec_i}$. These axioms can be verified to be meaningful by examining each axiom with the aid of Fig. 3 which shows poset events X and Y in two-dimensional space-time, and their proxies as per Definitions 4 and 5.

XP16. $R1a^{\prec_i} = R1a^{\prec}$.

XP17. $R1?^{\prec} \sqsubseteq R1?^{\prec_i}$, where ? is instantiated from $\{b, b', c, c', d\}$.

XP18. $R2a^{\prec_i} \sqsubseteq R2a^{\prec}$.

XP19. $R2b'^{\prec_i} = R2b'^{\prec}$, $R2b^{\prec_i} = R2b^{\prec}$.

XP20. $R2c^{\prec_i} \parallel R2c^{\prec}$, $R2c'^{\prec_i} \parallel R2c'^{\prec}$.

XP21. $R2d^{\prec} \sqsubseteq R2d^{\prec_i}$.

XP22. $R3a^{\prec_i} \sqsubseteq R3a^{\prec}$.

XP23. $R3b'^{\prec_i} \parallel R3b'^{\prec}$, $R3b^{\prec_i} \parallel R3b^{\prec}$.

XP24. $R3c^{\prec_i} = R3c^{\prec}$, $R3c'^{\prec_i} = R3c'^{\prec}$.

XP25. $R3d^{\prec} \sqsubseteq R3d^{\prec_i}$.

XP26. $R4?^{\prec_i} \sqsubseteq R4?^{\prec}$, where ? is instantiated from $\{a, b, b', c, c'\}$.

XP27. $R4d^{\prec} = R4d^{\prec_i}$.

The inclusion relationship of relations in $\mathcal{R}^{\prec}$ with respect to relations in $\mathcal{R}^{\prec_i}$, as captured in axioms XP16–XP27, is depicted in Fig. 4. In this figure, individual hierarchies within $\mathcal{R}^{\prec}$ and within $\mathcal{R}^{\prec_i}$, which are the same as in Fig. 2, are shown only partially. Observe that the mapping of relations in Table 1 [11] to relations in $\mathcal{R}$ (Table 4), as given in Section 2.3, is independent of whether the proxies used to derive $\mathcal{R}$ are defined by Definition 4 or by Definition 5.

4. Conclusion

We presented a hierarchy of relative timing relations between nonlinear events in a distributed system. The hierarchy of relations is complete using first-order predicate

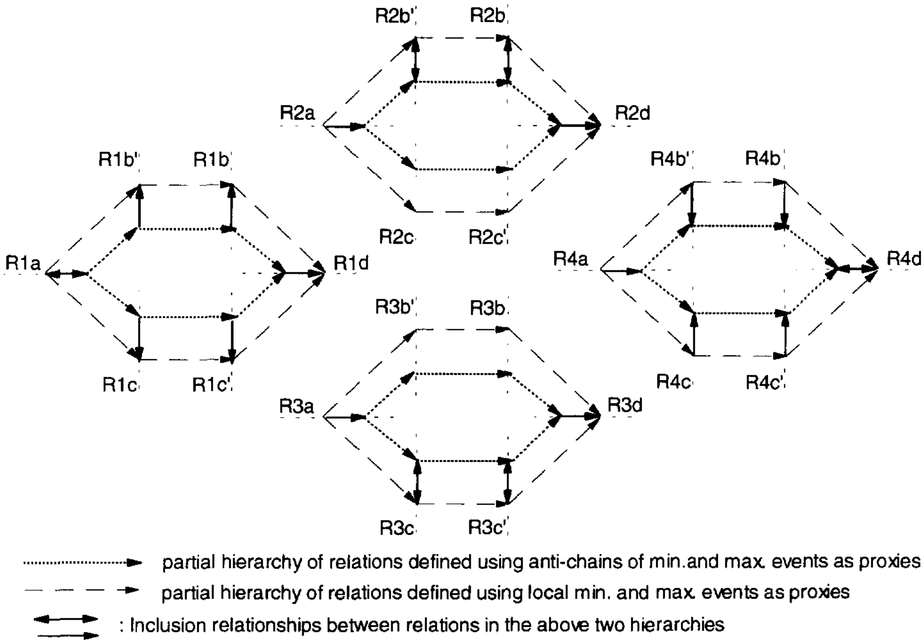

Fig. 4. Inclusion relationships between relations in hierarchies $\mathcal{R}^{\prec}$ and $\mathcal{R}^{\prec_i}$.

logic. We also presented an axiom system for reasoning intelligently with the proposed relations.

The results are useful in the study of time, temporal logic, and temporal and spatial representation and reasoning in distributed systems, as well as for applications which use nonatomicity in reasoning and modeling and need a fine level of discrimination of causality relations. Each application can choose appropriate causality relations from the exhaustive fine-grained hierarchy to specify and capture relative timing and synchronization conditions between its nonatomic poset events at a fine level of discrimination. This allows for a sophisticated modeling of the interactions in the application and system. The exhaustive classification gives an insight into the existing possibilities and can be used to select a number of primitive relations with good properties and clear intuitions. The axiom system on the relations enables reasoning with different levels of causality relations between nonatomic poset events. The axiom system is complete because the axioms can be used to derive all possible implications from any given valid predicates on the proposed relations.

The use of proxies in the definition of the proposed causality relations reduced the evaluation for causality between two nonatomic poset events X and Y from $|X| \times |Y|$ to $|N_X| \times |N_Y|$ tests for causality between atomic events in terms of which the two nonatomic events are defined. It is shown in [10] that the evaluation of the causality relations in Table 4 can by further simplified using properties of partial orders.

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