Solutions to remaining problems of Assignment #1

(Course: CS 401)

Problem 3: This question is related to two claims made by Professor *Smart*, who has also stated that he is smarter than the instructor and all the students in this class. We will examine his smartness by verifying the claims that he made.

Given an undirected, weighted graph G (with non-negative weights $w(u, v)$ for each edge $\{u, v\}$), and a constant $D > 0$, Professor *Smart* defines the graph G^{+D} as follows:

> G^{+D} *is identical to* G *except we* **add** *the constant* D *to each edge weight.*

(a) Professor *Smart* claims that:

> *a minimum spanning tree (MST) of* G *is* **always** *an MST of* G^{+D}.

(b) Professor *Smart* claims that:

> *a shortest path between vertices* s *and* t *in* G *is* **always** *a shortest path between* s *and* t *in* G^{+D}.

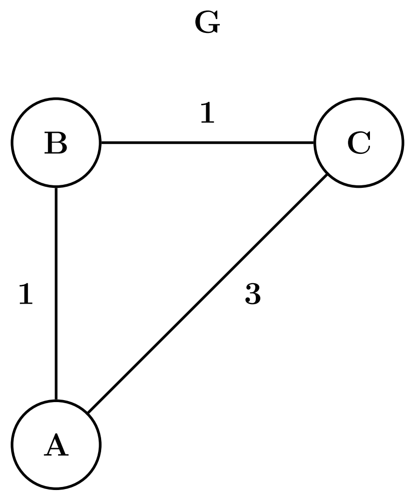
For *each* of the above two claims, your task is the following:

- If the claim is true, then provide a proof of the claim. This will show that Professor *Smart* was *indeed smart*.
- If the claim is false, provide a counter-example and explain why the counter-example shows that the claim is false. This will show that Professor *Smart* was *not so smart* after all.

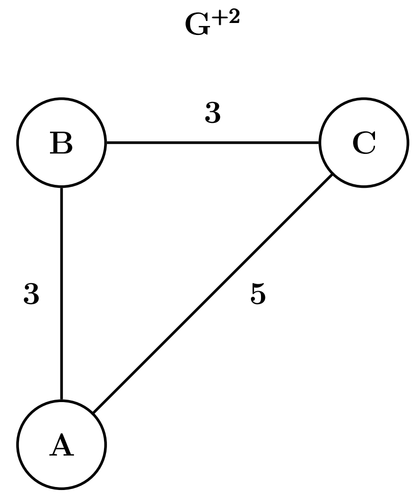
Solution:

(a) Yes, the claim is true. To see why this is true, consider Kruskal's algorithm for finding an MST. In this algorithm, we consider the edges in non-decreasing order of weights. Since the constant D is added to *every* edge, the ordering of edges by non-decreasing order of weights remains the same. So, Kruskal's algorithm will consider the edges in the same order and will generate the same MST.

(b) No, the claim is false. The following is a simple counter-example.



The shortest path between A and C is A–B–C of total weight 2



The shortest path between A and C is A–C of total weight 5

Problem 4: What is the total space required (in Θ notation) to represent a tree of n nodes using the adjacency list representation? Justify your answer.

Solution: The number of edges m in a tree of n nodes is exactly $n-1$. Thus, the total space taken is $\Theta(m+n) = \Theta(n)$.