Assignment #1
(Course: CS 401)

These are the first two problems for Assignment 1. The deadline is September 27, 2012, in class. No late assignments will be accepted.

The remaining problems of Assignment 1 will be given out later.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No hand waiving, vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 (40 points):
(a) [25 points] Take the following functions and arrange them in *descending* order of growth rates. That is, if a function $g_i(n)$ immediately follows function $g_j(n)$ in your list, then it should be the case that $g_i(n) = O(g_j(n))$. **Justify your answer clearly**.

$$g_1(n) = 2^{(\log n)^{1/3}}$$

$$g_2(n) = 2^{n^2}$$

$$g_3(n) = n^{5/3}$$

$$g_4(n) = \frac{\sqrt{\log n}}{\log \log n}$$

$$g_5(n) = n^{(\log n)^2}$$

$$g_6(n) = 2^{(11/9)^n}$$

$$g_7(n) = (\log n)^{\log n}$$

(b) [15 points] Give examples of two continuous functions $f(n)$ and $g(n)$ of positive real inputs n such that $f(n) \neq O(g(n))$, $g(n) \neq O(f(n))$, $f(n) \neq \Omega(g(n))$ and $g(n) \neq \Omega(f(n))$.

> Hint: You can specifiy some of the values of $f(n)$ and/or $g(n)$ depending on some condition on n, such as when n is even or odd, but make sure that your function is continuous and defined for all positive real n.

Problem 2 (20 points): Suppose that there are only two classifications of professional wrestlers: the "good wrestlers" and the "bad wrestlers". Further suppose that two wrestlers can wrestle only if one of them good and the other one is bad.

Your input is a list of n wrestlers and a list of r pairs of wrestlers that have rivalries. Your goal is to write an algorithm to determine, in $O(n+r)$ time, if it is possible to classify the n wrestlers as good or bad such that the two wrestlers in every rivalry pair can in fact wrestle.

For example, if the wrestlers are W_1, W_2, W_3 and the rivalry pairs are $\{W_1, W_2\}$, $\{W_1, W_3\}$, then classifying W_1 as good and W_2, W_3 as bad works. If, one the other hand, the rivalry pairs are $\{W_1, W_2\}$, $\{W_3, W_2\}$, $\{W_1, W_3\}$, then no matter how we classify the wrestlers, one rivalry pair will contain two wrestlers of the same type.