Solutions to Problem 3

(a) The graph on nodes $v_1, \ldots, v_5$ with edges $(v_1, v_2), (v_1, v_3), (v_2, v_5), (v_3, v_4)$ and (v_4, v_5) is such an example. The algorithm will return 2 corresponding to the path of edges (v_1, v_2) and (v_2, v_5), while the optimum is 3 using the path $(v_1, v_3), (v_3, v_4)$ and (v_4, v_5).

(b) The idea is to use dynamic programming. The simplest version to think of uses the subproblems $OPT[i]$ for the length of the longest path from v_1 to v_i. One point to be careful of is that not all nodes v_i necessarily have a path from v_1 to v_i. We will use the value "$-\infty$" for the $OPT[i]$ value in this case. We use $OPT(1) = 0$ as the longest path from v_1 to v_1 has 0 edges.

```
Long-path(n)
  Array M[1...n]
  M[1] = 0
  For i = 2,...,n
     M = -∞
     For all edges (j,i) then
        if M[j] ≠ -∞
             if M < M[j] + 1 then
                  M = M[j] + 1
             endif
        endif
     endfor
     M[i] = M
  endfor
  Return M[n] as the length of the longest path.
```

The running time is $O(n^2)$ if you assume that all edges entering a node i can be listed in $O(n)$ time.