Homework #2
(Course: CS 401)

This is the remaining third problem for Assignment 2. The deadline is November 27 (Tuesday), 2012, in class. No late assignments will be accepted.
Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No hand waiving, vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 3 **(**50 **points):** Let $G = (V, E)$ be a directed graph with nodes $v_1, v_2, \ldots, v_n$. We say that G is an **ordered graph** if it has the following properties.

(i) Each edge goes from a node with a lower index to a node with a higher index. That is, every directed edge has the form v_i, v_j with $i < j$.

(ii) Each node except v_n has at least one edge leaving it. That is, for every node v_i, $i = 1, 2, \ldots, n-1$, there is at least one edge of the form (v_i, v_j).

The *length* of a path is the number of edges in the path. The goal in this question is to solve the following problem

> **Given an ordered graph G, find the length of the longest path that begins at v_1 and ends at v_n.**
>
> (see Figure 1 for an example)

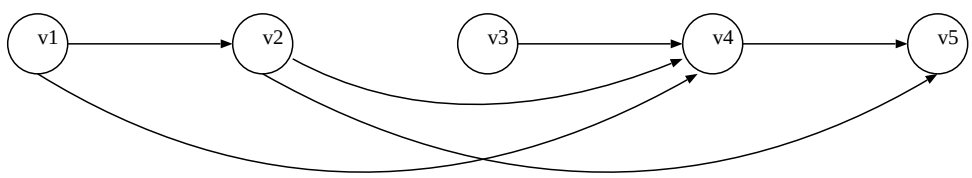

Figure 1: The correct answer for this ordered graph is 3: the longest path from v_1 to v_n uses three edges (v_1, v_2), (v_2, v_4), and (v_4, v_5)

(a) [20 points] Show that the following algorithm **does not** correctly solve this problem, by giving an example of an ordered graph on which it does not return the correct answer.

set $w = v_1$
set $L = 0$
while (there is an edge out of the node w) **do**
  choose the edge (w, v_j) for which j is *as small as possible*
  set $w = v_j$
  increase L by 1
end while
return L as the length of the longest path

In your example, say what the correct answer is and also what the above algorithm finds.

(b) [30 points] Give an efficient algorithm using dynamic programming approach that takes an ordered graph G and returns the length of the longest path that begins at v_1 and ends at v_n.