

Homework # 4

CS 594 – Applied Combinatorial Optimization

Due: Mon, Oct 21

Part I: Devise an LP formulation for the following problem. You are given a set of n tasks $\{t_1, \dots, t_n\}$ which have precedence constraints encoded in a DAG. There is a single start task t_1 (i.e., it has indegree of 0) and a single end task t_n (it has outdegree of 0). For a task to begin, all of its predecessors in the DAG must complete. If you know the task delays, the completion of the entire process is the length of the longest vertex-weighted path in the DAG. In this problem, you can spend money accelerate some of the tasks; your goal will be to achieve a particular process delay at minimum cost (this is a so-called PERT problem).

The additional input to the problem is as follows.

- A nominal delay d_i for each task t_i . This is how long the task takes if you spend no money on it.
- A minimum delay m_i for each task t_i . No matter how much money you spend on task i , it cannot complete in less than m_i time.
- A cost parameter c_i for task t_i . This is the amount of money you must pay to reduce the delay of the i th task by one time unit.
- A target delay for the entire system (the sink task t_n must complete by this time) T_{min} .

Part II: In many situations, our notion of cost c_i is not realistic – i.e., because of the principal of diminishing returns. For instance, it may cost one dollar to reduce the delay of a node by node minute, but it may take two dollars to reduce it an additional minute. Propose a reasonable formulation of the problem in this situation and show how to modify the LP accordingly. Note, we still have the m_i parameters.