CS201 Midterm Test

Name: SSN:

1. (7 marks) The relation R is a relation on the set {a, b, c, d}. Which one (ones) is (are) equivalence relation(s)?
   a) R={(a, a), (b, b), (c, c), (d, d)}
   b) R={(a, a), (a, b), (b, b), (b, a), (a, d), (a, c), (c, a), (c, c), (d, a), (d, d)}
   c) R={(a, a), (a, d), (b, b), (c, c), (d, a), (d, d)}
   d) R={(a, a), (a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, c), (c, d), (d, d)}

2. (7 marks) Each of following is a relation on {a,b,c,d}. Which one (ones) is (are) partial order(s)?
   a) R={(a, a), (b, b), (c, c), (d, d)}
   b) R={(a, a), (a, b), (b, b), (b, a), (a, d), (a, c), (c, a), (c, c), (d, a), (d, d)}
   c) R={(a, a), (a, c), (a, d), (b, b), (c, a), (c, d), (d, d)}
   d) R={(a, a), (a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, c), (c, d), (d, d)}

3. (10 marks) For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is anti-symmetric, and whether it is transitive. Put a tick in the table cell if you think that the corresponding relation has the particular property.

	Reflexive	Symmetric	Anti-symmetric	Transitive
(a) {(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4)}	x	x		x
(b) {(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)}			x	x
(c) {(1,3),(1,1),(3,1),(1,2),(3,3),(4,4),(3,2)}				x
(d) {(1,1),(2,2),(3,3)}		x	x	x

4. (10 marks) Fill in the table. Z is the set of integers and N is the set of natural numbers, 0, 1, 2, ….

	onto	one-to-one	bijection
$f : N \rightarrow N$, $f(x) = x + 1$		x	
$g : Z \rightarrow Z$, $g(x) = x + 1$	x	x	x
$h : Z \rightarrow N$, $h(x) = \lvert x \rvert$	x		
$m : N \rightarrow Z$, $m(x) = x*2$		x	

5. (10 marks) Let A be the set {2, 5, 7, 8, 12, 13} and B the set {1, 2, 3, 5, 8, 13, 14}. Compute:
   (a) $A \cap B$ = {2, 5, 8, 13}
   (b) A-B = {7, 12}
   (c) B-A = {1, 3, 14}

6. (10 marks) Let A = {1, 2, 5, 8, 11, {3}}. Identify each of the following as True or False.

	True	False
(a) $\{5, 1\} \subseteq A$	x	
(b) $\{8, 1\} \in A$		x
(c) $\{3\} \in A$	x	
(d) $\{\{3\}\} \subseteq A$	x	
(e) $3 \in A$		x

7. (8 marks) Given A = {1, 2, 9} and B = {2, 5, 6, 8}, Make True or False for each of the following:

	True	False
(a) $\{1, 2, 5\} \subset A - B$		x
(b) $A \cap B \subset P(B)$		x
(c) $A\text{-}B \in P(A)$	x	
(d) $\{\varnothing\} \subset P(A)$	x	

8. (10 marks) Fill the truth table of the following formula $\neg (p \vee \neg (p \wedge q))$

p	q	$p \wedge q$	$\neg (p \wedge q)$	$p \vee \neg (p \wedge q)$	$\neg (p \vee \neg (p \wedge q))$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	T	T	F

9. (18 marks) Let *p*, *q*, and *r* be propositions; *p* is known to be true, *q* is known to be false, and *r*'s status is unknown at this time. Tell whether each of the compound propositions is true, is false, or has unknown status.

	true	false	unknown
a) $p \vee r$	x		
b) $p \wedge r$			x
c) $p \rightarrow r$			x
d) $q \rightarrow r$	x		
e) $r \rightarrow p$	x		
f) $r \rightarrow q$			x
g) $(p \wedge r) \rightarrow r$	x		
h) $(p \vee r) \rightarrow r$			x
i) $(q \wedge r) \rightarrow r$	x		

10. (10 marks) Use mathematic induction to prove

    $n! \geq 2^{n-1}$ for $n \geq 1$

Answer:

Show that it is true for n=1: 1!= 1 , 2^{1-1}=1, $n! \geq 2^{n-1}$ is true for n=1.

Show that for all integers k >= 1, if the property is true for n=k then it is true for n=k+1

Suppose $k! \geq 2^{k-1}$, we will show that $(k+1)! \geq 2^{k+1-1} = 2^{k}$.

(k+1)!=k!*(k+1) $\geq 2^{k-1}$*(k+1) $\geq 2^{k-1}$*2= 2^{k}

Since we have proved both the base case and the inductive case, we conclude that the statement is true.