CS201 Quiz 1

Name: SSN:

1. (5 marks) Which of the following is logically equivalent to "I'm not taking umbrella or it's not raining"?
   (a) "If I'm taking umbrella, then it's raining."
   (b) "If it's raining, then I'm taking umbrella."
   (c) "If it's raining, then I'm not taking umbrella."
   (d) None of the above.

   Answer: (c)

2. (10 marks) Write the statement formally using quantifiers and variables, and write a negation for the statement.

   Every object is larger than some object.

   Answer:

   (1) $\forall x \in O, \exists y \in O,$ such that x is larger than y

   (2) $\exists x \in O,$ such that $\forall y \in O,$ x is not larger than y.

3. (10 marks) Translate the following sentences into propositional logic using the specified letter:

   S: Sally goes to the party.

   J: Jim goes to the party.

   (1) Sally goes to the party and Jim does not go to the party.

   Answer: $S \wedge \neg J$

   (2) If Jim goes to the party, Sally goes to the party.

   Answer: $J \rightarrow S$

   (3) Sally only goes to the party if Jim does not go.

   Answer: $S \rightarrow \neg J$

4. (7 marks) Write the contraposition, converse and inverse of the below conditional statement in English: (The statement may or may not be true.)

   If P is a rectangle, then P is a square.

   Answers:

   Contraposition: If P is not a square, then P is not a rectangle.

   Converse: If P is a square, then P is a rectangle.

   Inverse: If P is not a rectangle, then P is not a square.

5. (8 marks) Using a truth table to show that $p \to q$ is equivalent to $\neg p \vee q$.

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

6. (20 marks) Use mathematical induction to prove:

   (1) $8^n + 6$ is divisible by 14 for all positive integers n.

   Answer:

   Show that it is true for n=1: $8^n + 6 = 8^1 + 6 = 14$, 14 is divisible by 14.
   Show that for all integers k >= 1, if the property is true for n=k then it is true for n=k+1
   Assume that $8^k + 6$ is divisible by 14, we will show that $8^{k+1} + 6$ is also divisible by 14.
   Since $8^k + 6$ is divisible by 14, then $8^k + 6 = 14m$, where m is an integer, and $8^k = 14m-6$.
   We have
   $(8^{k+1}+6)=8*8^k+6 = 8(14m - 6) + 6= 8*14m - 42 = 14(8m - 3)$, which is divisible by 14.
   Since we have proved both the base case and the inductive case, we conclude that the statement is true.

   (2) $1+3n \leq 4n$ for every integer $n \geq 1$.

   Answer:

   Show that it is true for n=1: $1+3*1= 4$, $4*1=4$, $1+3n \leq 4n$ is true for n=1.
   Show that for all integers k >= 1, if the property is true for n=k then it is true for n=k+1
   Suppose $1+3k \leq 4k$, we will show that $1+3(k+1) \leq 4(k+1)$.
   $1+3(k+1) = 1+3k+3 \leq 4k+3 < 4k+4 = 4(k+1)$
   Since we have proved both the base case and the inductive case, we conclude that the statement is true.