Quiz 1(CS201)

Your Name: ___________________ **Your SSN: _________________**

Instructions

- This is a closed-book quiz.
- The quiz has 8 questions, and the full mark is 80.
- Write the answer for each question in the space provided below the question.

1. (10 marks) Is the following argument form *valid* or *invalid*? Construct a truth table and indicate which columns represent the premises and which the conclusion. Explain why it is *valid* or *invalid*.

$$p \to q \vee \sim r$$
$$q \to p \wedge r$$
$$\therefore p \to r$$

Solution

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	F
T	F	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Although there are several situations in which the premises and the conclusion are all true (rows 1, 7, and 8), there is one situation (shown in row 4) where the premises are true and the conclusion is false. This cannot occur when the argument form is valid, and so this argument form is invalid.

2. (10 marks) Use the laws of logical equivalences (do not use truth tables) to show the following equivalence:

$$A \wedge \neg(A \wedge B) \equiv A \wedge \neg B$$

Supply a reason for each step.

Solution

$$A \wedge \neg(A \wedge B) \equiv A \wedge (\neg A \vee \neg B) \qquad \textit{by De Morgan's laws}$$
$$\equiv (A \wedge \neg A) \vee (A \wedge \neg B) \qquad \textit{by the distributive laws}$$
$$\equiv c \vee (A \wedge \neg B) \qquad \textit{by the negation laws}$$
$$\equiv (A \wedge \neg B) \qquad \textit{by the identity laws}$$

3. (10 marks) Write the meaning of each statement in English. State whether it is true or false. The universe of discourse in each case (for both x and y) is the set of all integers greater than or equal to 0.

   a. $\forall x \exists y (x < y)$

      Solution

      The given statement says that given any integer x (≥ 0), there is an integer y (≥ 0) that is greater than x. This is true: For any integer x, let $y = x + 1$, then $x < y$.

   b. $\forall y \exists x (x < y)$

      Solution

      The given statement says that given any integer y (≥ 0), there is an integer x (≥ 0) that is less than y. This is false: when $y = 0$, there is no x (≥ 0) that satisfies $x < y$.

   c. $\exists x \exists y (x < y)$

      Solution

      Some integer x is less than some integer y. This is true: e.g. $x = 1$ is less than $y = 2$.

4. (10 marks) Rewrite the following statement in predicate logic.

   a. Every computer science student needs to take CS201.

      Solution

      $\forall x$, if x is a computer scientist, then x needs to take CS201. or
      $\forall$ computer scientists x, x needs to take CS201.

   b. Somebody likes every book.

      Solution

      $\exists$ a person x such that $\forall$ books y, x likes y.

   c. No math course is easy

      Solution

      $\forall$ math courses x, x is not easy.

5. (8 marks) If n is an integer that is greater than 1, then

   n is prime $\Leftrightarrow$ $\forall$positive integers r and s, if $n = r \cdot s$ then $r = 1$ or $s = 1$.

   n is composite(not prime) $\Leftrightarrow$ $\forall$positive integers r and s such that $n = r \cdot s$ and $r \neq 1$ and $s \neq 1$.

   Prove: $n^2 + 3n + 2$ is not prime.

   Solution

   Suppose n is any positive integer. We can factor $n^2 + 3n + 2$ to obtain $n^2 + 3n + 2 = (n + 1)(n + 2)$. We also note that $n + 1$ and $n + 2$ are integers (because they are sums of integers) and that both $n + 1 > 1$ and $n + 2 > 1$ (because $n \geq 1$). Thus $n^2 + 3n + 2$ is a product of two integers each greater than 1, and so $n^2 + 3n + 2$ is not prime.

6. (12 marks) Recursion

a. Recursively define the sequence: 5, 10, 15, 20, 25, …

Solution

$S(1) = 5$

$S(n) = S(n-1) + 5$ for $n \geq 2$

b. Give f(3), f(4) and f(5) for the following recursively defined sequence

$f(1) = 1, f(2) = 2$

$f(n) = f(n-1)*f(n-2) + 1 \quad$ for $n > 2$

Solution

$f(3) = f(2)*f(1) + 1 = 2*1 + 1 = 3$

$f(4) = f(3)*f(2) + 1 = 3*2 + 1 = 7$

$f(5) = f(4)*f(3) + 1 = 7*3 + 1 = 22$

d. Give the result of foo(4, 4) for the following program

```
float foo(float x, int n)
{
    if (n==0)          /* if (n==0) */
        return 0
    else
        return x + foo(x, n-1)
}
```

Solution

$$
\begin{aligned}
Foo(4,4) &= 4 + foo(4,3) \\
&= 4 + (4 + foo(4,2)) \\
&= 4 + (4 + (4 + foo(4,1))) \\
&= 4 + (4 + (4 + (4 + foo(4,0)))) \\
&= 4 + (4 + (4 + (4 + 0))) \\
&= 16.
\end{aligned}
$$

7. (10 marks) Use mathematical induction to prove

$$\sum_{i=1}^{n}(2i-1)=n^2$$

Proof

For the given statement, the property is the equation $\sum_{i=1}^{n}(2i-1)=n^2$

Show that the property is true for *n* = 1:

When $n = 1$, the left-hand side of the equation is 2*1-1 = 1. And the right-hand side is 1^2 = 1 also. Thus the property is true for $n = 1$.

Show that for all integers *k* ≥ 1, if the property is true for *n* = *k* then it is true for *n* = *k* + 1:

Let k be any integer with $k \geq 1$, and suppose the property is true for $n = k$. That is, suppose

$$\sum_{i=1}^{k}(2i-1)=k^2$$

We must show that the property is true for $n = k + 1$. That is, we must show that

$$\sum_{i=1}^{k+1}(2i-1)=(k+1)^2$$

But the left-hand side of equation is

$$\begin{aligned}
\sum_{i=1}^{k+1}(2i-1) &= \sum_{i=1}^{k}(2i-1)+\{2(k+1)-1\} \\
&= k^2+\{2(k+1)-1\} && \textit{by substitution from the inductive hypothesis} \\
&= k^2+2k+1 \\
&= k^2+k+k+1 \\
&= k(k+1)+(k+1) \\
&= (k+1)(k+1) \\
&= (k+1)^2
\end{aligned}$$

and this is the right-hand side of equation. Hence the property is true for $n = k + 1$.

8. (10 marks) Using mathematical induction to prove: For any positive integer n, $n^2 + n$ is divisible by 2.

Proof

Let the property be the sentence "$n^2 + n$ is divisible by 2."

Show that the property is true for $n = 1$:

To show the property is true for $n = 1$, we must show that $1^2 + 1 = 2$ is divisible by 2. But this is true because $2 = 2 \cdot 1$.

Show that for all integers $k \geq 1$, if the property is true for $n = k$ then it is true for $n = k + 1$:

Suppose $k^2 + k$ is divisible by 2, for some integer $k \geq 1$.

By definition of divisibility, this means that

$$k^2 + k = 2r \qquad \text{for some integer } r.$$

We must show that $(k + 1)^2 + (k + 1)$ is divisible by 2.

But

$$\begin{aligned}(k + 1)^2 + (k + 1) &= k^2 + 2k + 1 + (k + 1) \\ &= k^2 + 2k + k + 2 \\ &= (k^2 + k) + 2k + 2 \\ &= 2r + 2k + 2 && \text{by inductive hypothesis} \\ &= 2(r + k + 2)\end{aligned}$$

But $(r + k + 2)$ is an integer because it is a sum of integers, and so, by definition of divisibility, $(k + 1)^2 + (k + 1)$ is divisible by 2.