CS201 Quiz 2 (Fall 2004)

Name: University ID:

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Instructions

- This is a closed-book quiz.
- The quiz has 10 questions, and the full mark is 100.
- Write the answer for each question in the space provided below the question.

1. (30 marks – 2 each) Let A = {1, 2, 3, 5, 8, 11, {3}, ∅}. Identify each of the following as True or False. P(X) means that power set of X.

	True	False
(1) {5, 11} ⊆ A	√	
(2) ∅ ∈A	√	
(3) {3}∈A	√	
(4) {3} ⊆ A	√	
(5) {{3}} ∈ A		√
(6) ∅ ⊆ A	√	
(7) {1, 6} ⊆ A		√
(8) {1, {2}} ∈ P(A)		√
(9) {∅, {3}} ∈ P(A)	√	
(10) ∅ ⊆ P(A)	√	
(11) {∅} ∈ P(A)	√	
(12) ∅ ∈ P(A)		√
(13) {3} ∈ P(A)	√	
(14) {1, {3}} ∈ P(A)	√	
(15) {1, {3}} ⊆ P(A)		√

2. (6 marks – 2 each) For any two sets A and B, if B ⊂ A, then

(a) $A \cup B = \mathrm{A}$

(b) $A \cap B = \mathrm{B}$

(c) B – A = ∅

3. (10 marks – 2 each) Let A = {2, 3, 4, 5, 7, 9} and B = {1, 2, 5, 6, 8, 10}. Compute:

   (a) $A \cap B = \{2, 5\}$

   (b) $A - B = \{3, 4, 7, 9\}$

   (c) $B - A = \{1, 6, 8, 10\}$

   (d) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   (e) $P((A-B) \cap B) = \{\varnothing\}$

4. (4 marks – 2 each) Draw a Venn diagram to find the sets A and B, if A – B = {4, 5}, B – A ={6, 7}, and $A \cup B = \{3, 4, 5, 6, 7, 8\}$.

   Answer: A = {4, 5, 3, 8}
   B = {3, 6, 7, 8}

5. (10 marks) Proofs:

   (a) (5 marks) Are the following two sets equal? If they are equal, prove it using mutual inclusion. If they are not equal, find a counterexample.

   $A = \{x \mid x \in Z_+ \text{ and } x = 3m \text{ for } m \in Z_+\}$

   $B = \{x \mid x \in Z_+ \text{ and } x = 3n+6 \text{ for } n \in Z_+\}$

   Answer: They are not equal. For example, $3 \in A$, but $3 \notin B$.

   (b) (5 marks) Are the following two sets equal? If they are equal, prove it using mutual inclusion. If they are not equal, find a counterexample.

   $A = \{x \mid x \in Z \text{ and } x = 3m \text{ for } m \in Z\}$

   $B = \{x \mid x \in Z \text{ and } x = 3n+6 \text{ for } n \in Z\}$

   Answer: They are equal.
   Proof sketch: (1) assume $x \in A$, $x = 3m = 3(m-2) + 6$. Since $m-2 \in Z$, we can use $m-2 = n$, thus, $x \in B$.
   (2) assume $x \in B$, $x = 3n+6 =3(n+2)$. Since $n+2 \in Z$, we can use $n+2 = m$, thus $x \in A$

6. (10 marks – 2 each) Answer for the following questions:

   (a) A small town has 300 residents. Must there be 2 residents who have the same birthday.

   No.

   (b) Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. Suppose 5 integers are chosen from S. Must there be two integers whose sum is 12?

   No.

   (c) Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. Suppose 6 integers are chosen from S. Must there be two integers whose sum is 12?

   No.

   (d) How many integers must you pick in order to be sure that at least two of them have the same remainder when divided by 11?

12.

(e) How many people must be in a group in order to be sure that at least two people have exactly the same first and last initials?

26x26+1 = 677

7. (10 marks – 2 each) ρ is a relation on set {a, b, c, d}. Which one (ones) is (are) equivalence relation(s)? No mark will be given if you mark everyone or you do not mark anyone.

a) ρ = {(a, a), (b, b), (c, c), (d, d)} Y

b) ρ = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, c), (c, a), (a, d), (d, a)} N

c) ρ = {(a, a), (b, b), (c, c), (d, d), (a, d), (d, a)} Y

d) ρ = {(a, a), (b, b), (c, c), (d, d), (b, c), (c, b), (c, d)} N

e) ρ = {(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (b, a), (b, d), (c, b)} N

8. (10 marks – 2 each) Each of following is a relation on {a, b, c, d}. Which one (ones) is (are) partial order(s)? No mark will be given if you mark everyone or you do not mark anyone.

a) ρ = {(a, a), (b, b), (c, c), (d, d)} Y

b) ρ = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, d), (a, c), (a, a), (d, a)} N

c) ρ = {(a, a), (b, b), (d, d), (a, c), (a, d), (c, a), (c, d)} N

d) ρ = {(a, a), (b, b), (c, c), (d, d), (b, c), (b, d), (c, d)} Y

e) ρ = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (a, c), (a, d), (b, d), (c, d)} Y

9. (2 marks) Given the following equivalence relation {a, b, c, d, e}, find the equivalence class of c, [c].

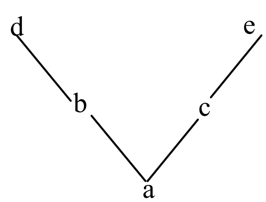
ρ = {(a, a), (b, b), (c, c), (d, d), (e, e), (c, d), (c, e), (d, c,), (d, e), (e, c), (e, d)}

Answer: [c] = {c, d, e}

10. (8 marks) Given the following partial order on {a, b, c, d, e}

ρ = {(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (c, e), (b, d), (a, c), (a, d), (a, e)}

a) (4 marks) Draw the Hasse diagram of relation ρ.



b) (4 marks) Give maximal elements, minimal elements, least element and greatest element, if any exists? If anyone of them does not exist, indicate it with a "no".

minimal elements: a

maximal elements: d, e

least element: a

greatest element: No.