Spring 2014 CS201 Homework 2

Due 02/01/2014 11:59 pm

**Q1 (10 points)**

For each positive integer n, let P(n) be the formula



a. Write P(1). Is P(1) true?

b. Write P(k).

c. Write P(k + 1).

d. In a proof by mathematical induction that the formula holds for all integers n ≥ 1, what must be shown in the inductive step?

**Q2 (10 points)**

Use the formula for the sum of the first n integers and/or the formula for the sum of a geometric sequence to evaluate the sums or to write them in closed form.

a. 3 + 4 + 5 + 6+· · ·+1000

b.

Where n is a positive integer.

**Q3 (50 points)**

Prove each of the statements by mathematical induction.

a.



for all integers n ≥ 2.

b.



for all integers n ≥ 1.

c. If x is a real number not divisible by π, then for all integers n ≥ 1,



d. 32n - 1 is divisible by 8, for each integer n ≥ 0.

e. 2n < (n + 1)!, for all integers n ≥ 2.

f. A sequence a1, a2, a3, . . . is defined by letting a1 = 3 and ak = 7ak-1 for all integers k ≥ 2. Show that an = 3·7n-1 for all integers n ≥ 1.

**Q4 (10 points)**

find the mistakes in the “proof” by mathematical induction.

“Theorem:” For any integer n ≥ 1, all the numbers in a set of n numbers are equal to each other.

“Proof (by mathematical induction): It is obviously true that all the numbers in a set consisting of just one number are equal to each other, so the basis step is true. For the inductive step, let A = {a1, a2, . . . , ak , ak+1} be any set of k + 1 numbers. Form two subsets each of size k:

B = {a1, a2, a3, . . . , ak } and

C = {a1, a3, a4, . . . , ak+1}.

(B consists of all the numbers in A except ak+1, and C consists of all the numbers in A except a2.) By inductive hypothesis, all the numbers in B equal a1 and all the numbers in C equal a1 (since both sets have only k numbers). But every number in A is in B or C, so all the numbers in A equal a1; hence all are equal to each other.”