Paper Presentation:

It's fourth down and what does the Bellman equation say? A dynamic-programming analysis of football strategy.

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Overview

- Rules of Football
- Fourth Down Situations
- Formal Analysis
- Results of Analysis

Relevant Rules of Football

- 100 Yard Football field
  - Center field is 50-yard line
  - Yard lines numbered 1 to 49 towards center field
  - Goal lines / End zones
- Four 15 minute quarters (with kickoff after half time)
- Game begins with kickoff: kickoff from *own* 30 yard line

Relevant Rules of Football (cont.)

- Receiving team has four downs (tries) to advance ball 10 yards (or score). If ball is advanced 10 yards then the team with possession receives a new set of downs.

- Scoring

  – Touch Down (6 pts plus extra point)

  – Field Goal (3 pts)

  – Safety (2 pts)- must then kick off to scoring team (adding insult to injury)

Fourth Down Decision

On fourth down we have a decision to make - we can:

- Attempt another play to obtain a new set of downs and 'go for it'
  - (+) We still have the ball and the chance to score
  - (-) If we do not make it then the opponent has possesion on the spot.

Fourth Down Decision (cont.)

- Punt (kick) the ball to the opponent.
  - (+) Typically moves the ball towards the opponents side of the field (on average 38 yards net)
  - (-) The opponent now has the chance to score

- Attempt to kick a field goal.
  - (+) Possibility of 3 pts
  - (-) Possibility of 3 pts (instead of 7)

What are we doing?

- In AI:
  - Explicitly model the evironment (states, actions, transition probabilities, immediate rewards)
  - Calculate an optimal solution (plan)
- In econometrics, data mining, OR:
  - Determine the model from the environment (data)
  - Determine if we are behaving in an optimal fasion

Data

- Use data from all first quarter situations from 1998-2000.

- 11,112 first quarter situations

  - kickoffs; 1,851 cases

  - first and 10 on own 20; 557 cases

- We will look at the *value of first down situations* as they result from 'going for it' and from kicking.

The Value of Situations

Lets consider 101 different types of situations:

- First and ten on each yard line from a team's one to it's opponent's 10

- First and goal on each yard line from opponent's 9 to it 1

- Kickoff from own 30 (following a score)

- Kickoff from own 20 (following a safety)

Notation

We will index the types from 1 to 101 and refer to the value of a particular situation i as $V(i)$ where $1 \leq i \leq 101$.

Let $S = \{s_i \in \mathbb{Z} : 1 \leq s_i \leq 101\}$, $s_s \in S$ denotes a starting situation, $s_e \in S$ denotes an ending situation, $p \in \mathbb{Z}$ denotes net points scored before the start of the next situation (s_e). Let $b \in \{1, -1\}$; $b = 1$ denotes that the team with possesion in s_s maintains possesion in s_e.

Then let an event be a four tuple: (s_s, s_e, p, b). Let E be the set of all events.

Preliminaries

- Let $\widehat{P}_i \in \mathbb{R}$ be the average net points scored in situation i *before* the start of the next situation (ex. could be negative if opponent scores more often) draw!.

- $\Pr(b = 1|s_s = i, s_e = j)$ is the probability that the team with possesion at time t in situation i maintains possesion at time $t + 1$ in situation j. $1 - \Pr(b = 1|s_s = i, s_e = j)$ is the probability of a turnover given i and j.

Bellman Equation

The value of states (situations) can be expressed as the "short term reward" plus the "expected long term reward":

$$U'(s_s) = R(s_s) + \gamma \max_a \sum_{s_e \in S} \Pr(s_e | a, s_s) U(s_e)$$

$R(s_s)$: the immediate expected average reward for being in state (situation) s_s.

Assume there is only one action; to play (make decisions) as the average NFL team in first quarters from 1998-2000, therefore we will drop the notion of an action.

Drop γ: we would not expect the value of a situation to be above 7 (bounded).

Value of Situations

A situation can be for example having a first down on *your own* 45 yard line. A possible next situation is having a first down on your *opponents* 40 yard line (we have advanced the ball 15 yards from our previous situation).

However *if we do not have posession in this next situation* then it is still a situation of a first and 10 on your opponents 40 yard line - however it is our opponent who has possesion (on our 40 yard line).

The value of the situation is still the same, however, if the opponent has possesion *then this is the value to the opponent*. Any value to our opponent is a penalty for us therefore we would subtract the values (weighted with associated probabilities) from our own value for situation j.

Computing the Value of Situations

The value of situation i is the average expected immediate value of situation i plus the expected value of future situations. Initially: $\forall i, V(i) = 0$

$$V'(i) = \widehat{P}_i + \sum_{j=1}^{101} \mathrm{Pr}(s_e = j | s_s = i) \ [\mathrm{Pr}(b = 1 | s_s = i, s_e = j) V(j) + \mathrm{Pr}(b = -1 | s_s = i, s_e = j)(-1) V(j)]$$

Which can be expressed as:

$$V'(i) = \widehat{P}_i + \sum_{j=1}^{101} \mathrm{Pr}(s_e = j | s_s = i) \ [2 \times \mathrm{Pr}(b = 1 | s_s = i, s_e = j) - 1] V(j)$$

Example

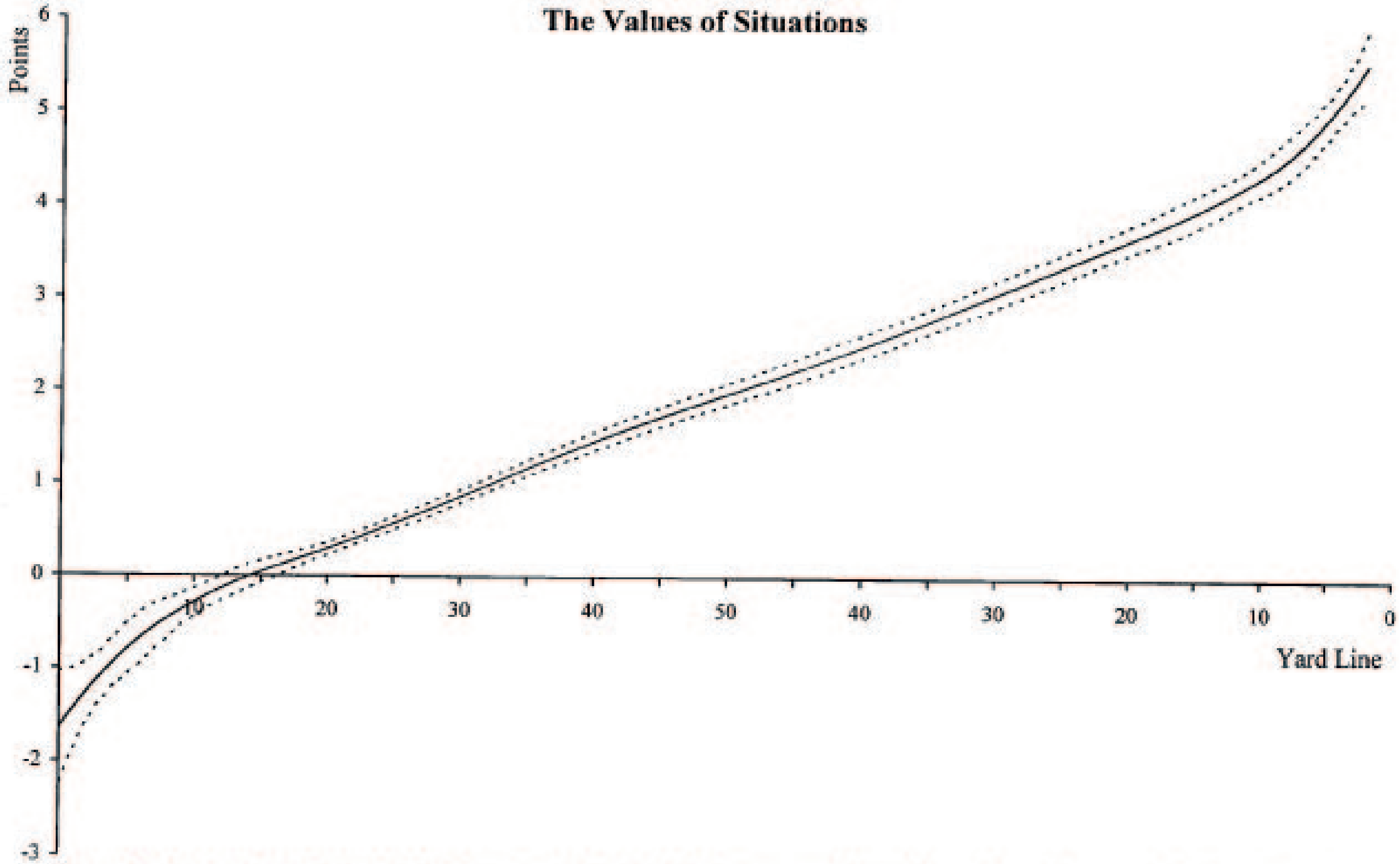
Consider the value of situation j is 6 and 75% of the time we had possesion in situation j, given that we started in situation i. Then the expected utility is: $(.75)(6)+(.25)(-1)(6) = 4.5-1.5 = 3.0$, which is $[2(.75)-1](6)$. This would be wieghted by the probability of situation j given that we are in situation i ($\Pr(s_e = j|s_s = i)$).

If we never had possesion in situation j given that we started in situation i then the contribution to the value of situation i is $[2 \times (0) - 1](6) \times \Pr(s_e = j|s_s = i) = -6 \times \Pr(s_e = j|s_s = i)$.

If we always had possesion then: $[2 \times (1.0) - 1](6) \times \Pr(s_e = j|s_s = i) = 6 \times \Pr(s_e = j|s_s = i)$.

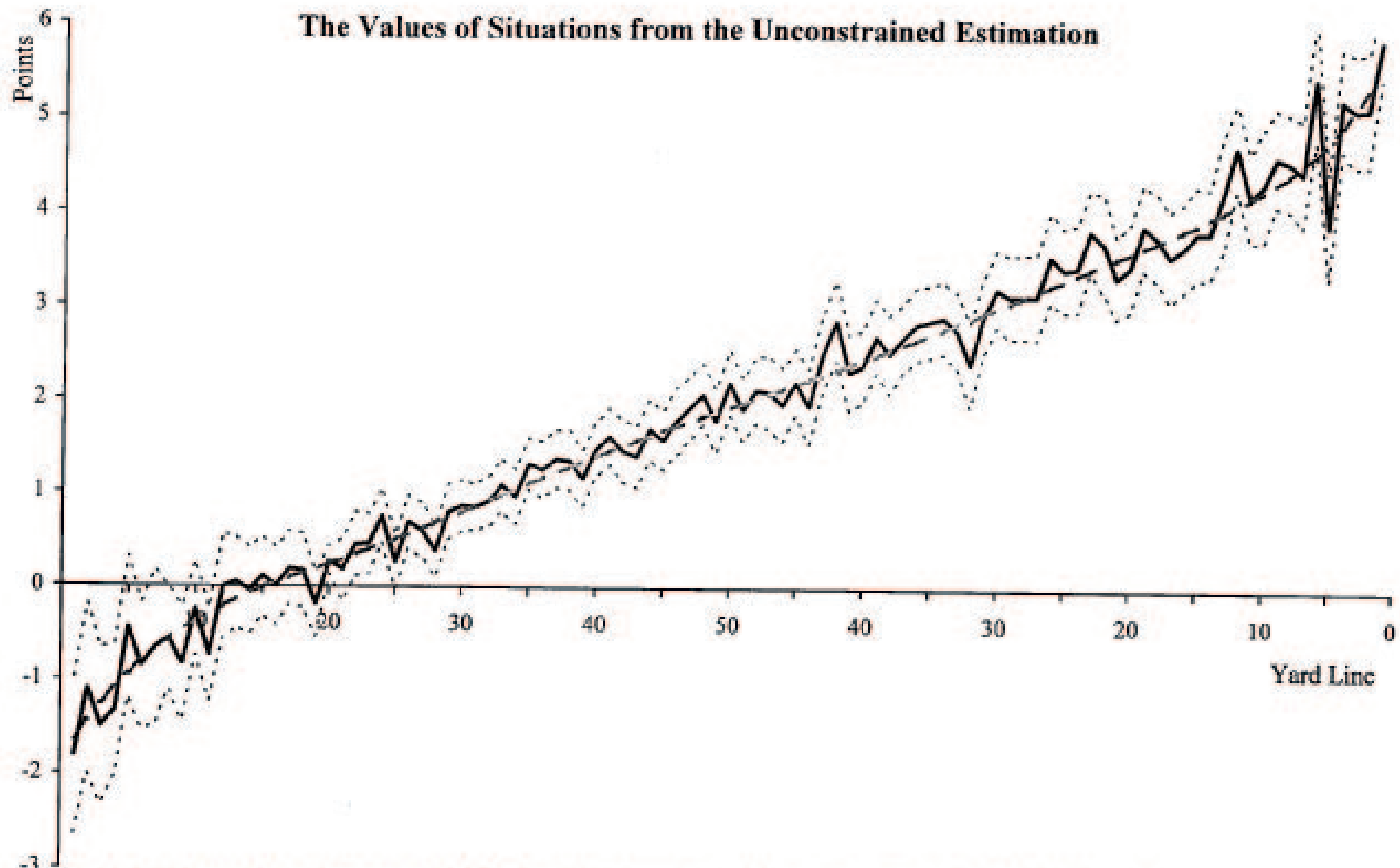
Figure 1

The Values of Situations



The solid shows the estimates of the V's from the splined estimation. The dotted lines show the two-standard-error bands. The estimated value of a kickoff is -0.62 (with a standard error of 0.04); the estimated value of a free kick is -1.21 (0.51).

Figure 2



The figure shows the estimates of the V's from the unconstrained estimation. The dotted lines show the two-standard-error bands. The dashed line shows the estimates of the V's from the splined estimation.

Comments on the value of situations

- Kickoff has a value -0.6 so the net value of a field goal is 2.4, the net value of a touchdown is 6.4

- Value of first and 10 on own 1 is -1.6 pts.

- Teams should be indifferent between a first and 10 on own 15 vs. having opponent in same situation.

- The value of recieving a kickoff is as valuable as a first and 10 on own 27 (+0.6).

Kicking vs. Going for it

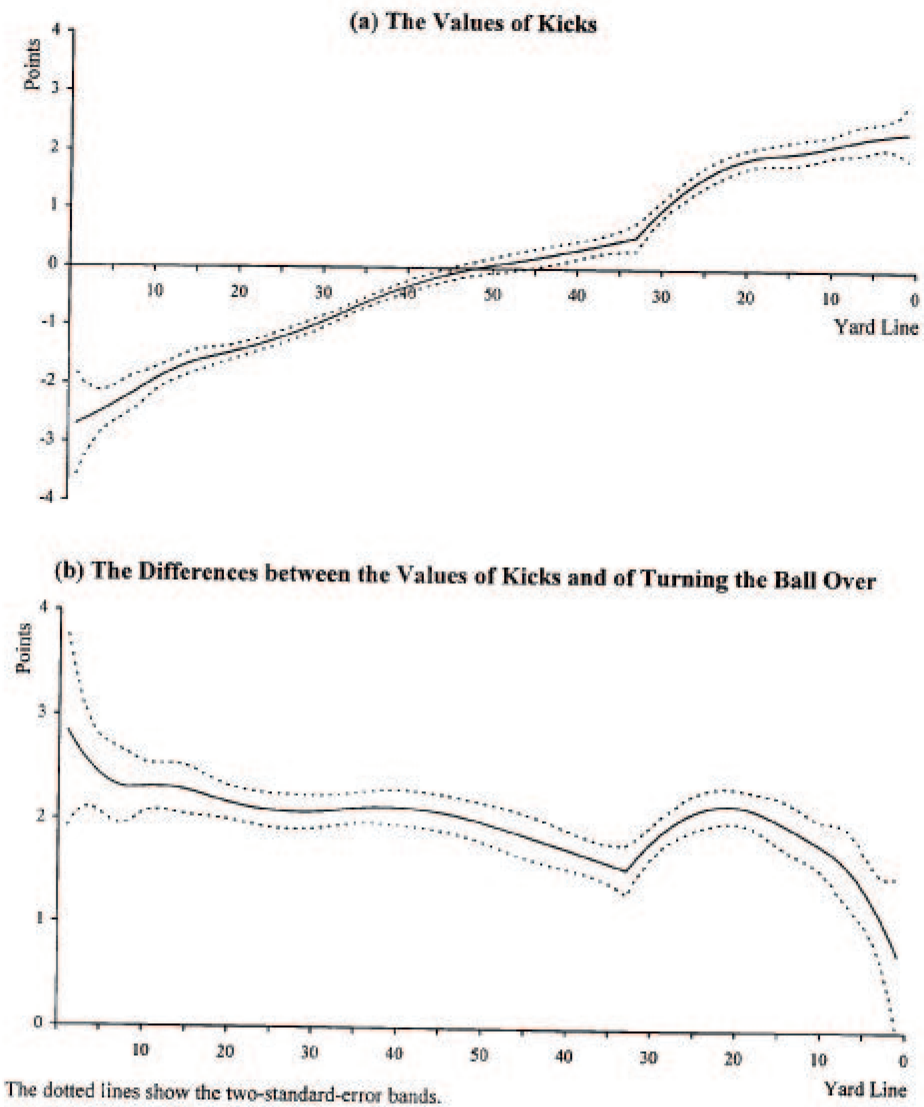
- Consider the net points scored before the next situation

- The next situations value

- 2,560 observations

- Note that the difference between the value of a kick and the value of turning the ball over drops as we get close to the opponents goal line.

Figure 3



The dotted lines show the two-standard-error bands.

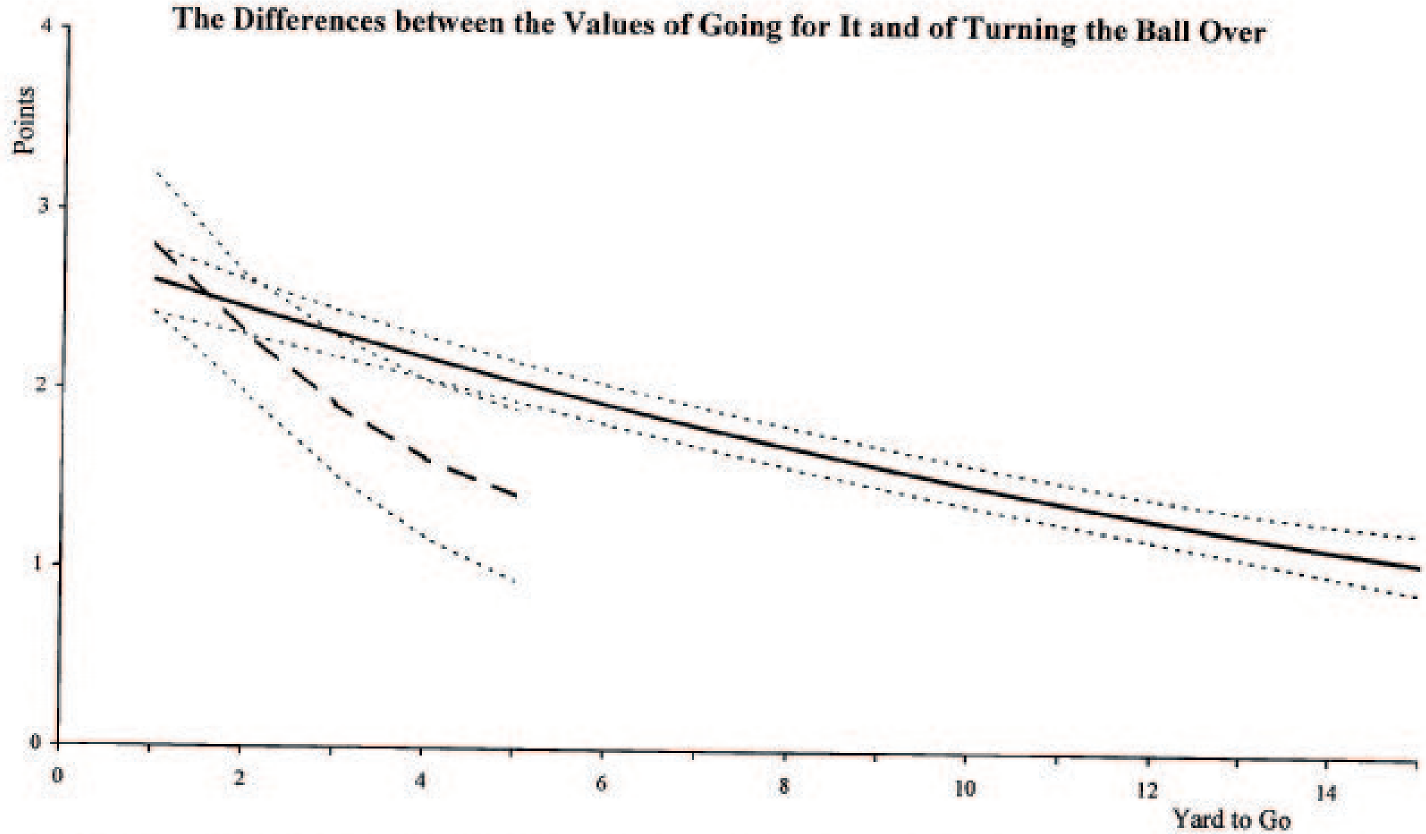
Value of going for it

- Use third down plays to estimate the likelihood of converting (50% of the time we will gain 6 yards)

- Value of going for it depends both on field position and yards to go for a first down or touchdown

- So the difference between the value of going for it and turning the ball over . . .

Figure 4



The figure shows the estimated differences between the values of going for it and the other team having the ball on the spot at a generic yard line outside the opponent's 17 (solid line) and at the opponent's 5 (dashed line). The dotted lines show the two-standard-error bands.

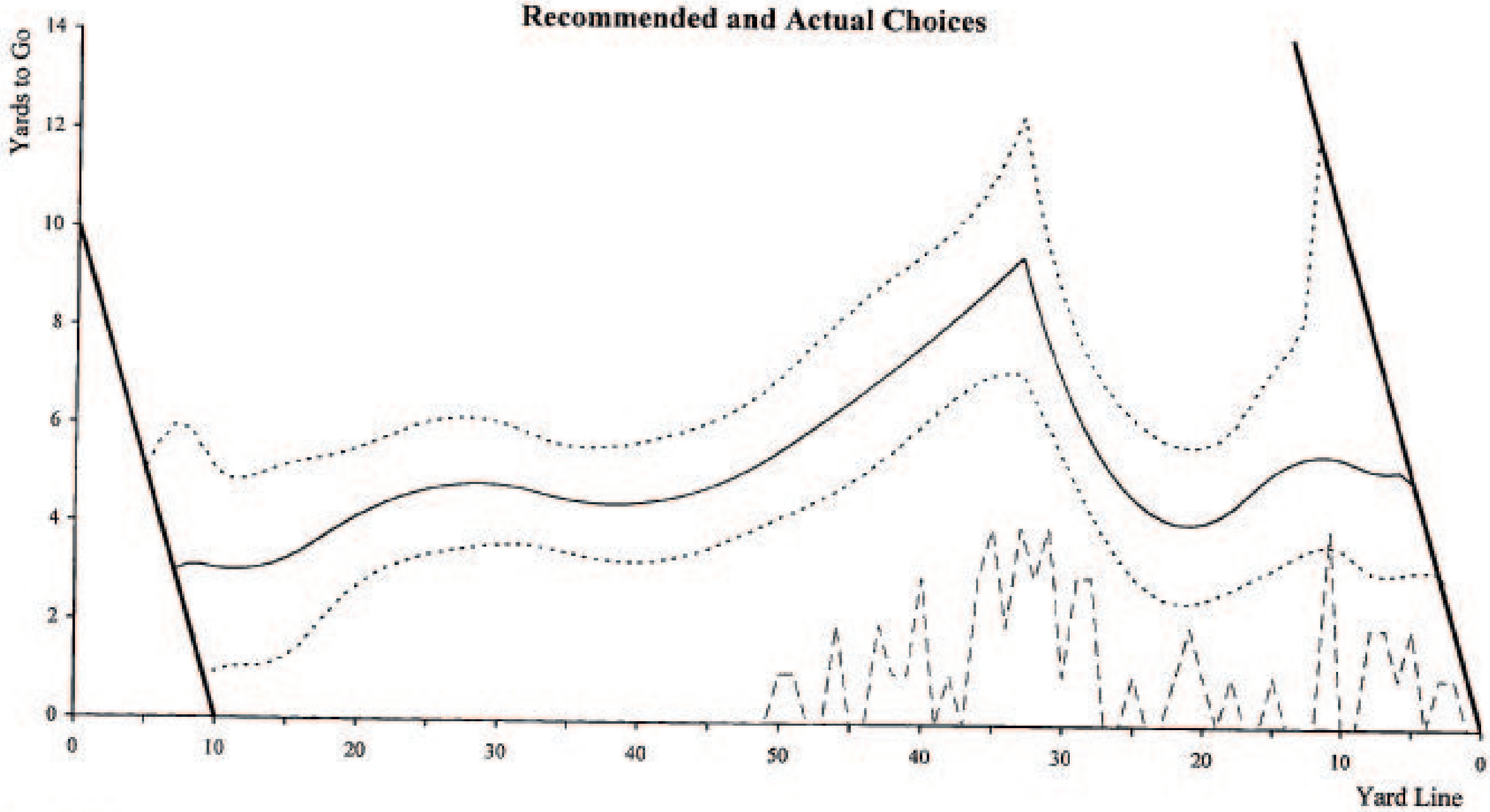
Reccommended Choices

- On own side of field with less than 4 yards to go; go for it
- Inside opponents 5 yard line; always go for it

Example: Fourth and goal on opponents 2; Going for it gives a 3/7 chance of success. Therefore a field goal and going for it have about the same expected immediate payoff. However, going for it leaves opponent with first and 10 on their 2 (4/7 chance) vice receiving a kickoff (their own 27).

Figure 5

Recommended and Actual Choices



The solid line shows the number of yards to go where the estimated values of kicking and going for it are equal. The dotted lines show the two-standard-error bands. The dashed line shows the greatest number of yards to go such that when teams have that many or fewer yards to go, they go for it at least as often as they kick.

Actual Choices

- On 1,575 fourth downs where analysis implies kicking; teams kicked 1,569 times (looks good)

- On 1,100 fourth downs where analysis implies going for it; teams went for it 108 times (?)

- Better decisions on fourth down (in the first quarter) could...
  - increase a teams probability of winning by 0.6%
  - allow a typical team to win one more game in three seasons out of four

Why?

The rewards for winning in the NFL are enormous. Why do teams appear to not maximize their chances of victory?

- They do- but they do so imperfectly
  - intuition
  - experience

Why? (cont.)

The rewards for winning in the NFL are enormous. Why do teams appear to not maximize their chances of victory?

- The costs of losing due to a failed gamble could be greater than the costs of losing from playing safe.
  - criticism and reduced support from fans
  - adverse consequences from decision makers higher in the organization
  - regret

Questions?