Research Summary

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1 Thesis Research

My thesis research is titled *Bargaining under Higher Order Uncertainty* and can be classified under the topic *Interactive Epistemology in Game Theory and Decision Theory.*

Broadly speaking, my work is motivated by the interactive epistemological approach to both game theory ([19], [21], [7], [2], [3], [5] and [28]) and decision theory ([11] and [12]). I am interested in developing *computationally tractable* algorithms for rational agents (Chapter 2 of [26]) that have to make decisions and plan while interacting sequentially in uncertain environments possibly containing other agents with possibly conflicting interests and goals. In particular, I have situated all my efforts pertaining to this research problem in developing models and algorithms for one concrete object of study – *bilateral bargaining under uncertainty.*

On the one hand, I study *sequentially rational* [20] and *Bayesian* (i.e. perfect Bayesian) equilibrium (PBE) solutions that are available for two-persn bargaining games under uncertainty ([24], [25], [27], [9], [17], [18], [22] and [8]) – i.e. where one or both of the players (throughout, 'player' is used interchangeably with 'agent') are incompletely informed about one or more payoff-relevant paramaters and represent such uncertainty using probabilistic beliefs. In the course of my research in this area, I was able to extend some classical results to settings where the *beliefs* of the player(s) are *not asummed to be commonly known*; instead, players' uncertainties about the others' uncertainties are expressed as higher order beliefs (using higher order probability distributions) up to some finite level.

In a paper under preparation, I construct pooling PBE for games with second and third order uncertainties (expressed using second and third order beliefs). Also, I report useful sensitivity results under this solution concept for games where I study the sensitivity of the expected payoffs, for each type of the player(s), to changes in the discount factor, the time horizon and the degree of uncertainty. The typical multiplicity of PBE in these games lead me to explore epistemic conditions where one equilibrium (or, class of equilibria) may be more compelling than another. I also explore the applicability of equilibrium refinements in these settings.

The second aspect of my research involves attacking the problem from a purely decision-theoretic (i.e. subjective expected utility maximization) standpoint. Here, instead of searching for equilibria, the agents maintain beliefs about the opponents type and then choose actions that maximizes its expected utility in the resulting interaction. The type of an agent includes its payoff-relevant attributes (for e.g., the player's valuation of an item in a bilateral bargaining game) and its beliefs. Beliefs about an opponent's type includes beliefs about the opponent's beliefs – this leads to a recursive nesting of the agents' beliefs which may, in principle, be infinite. This is dealt with by allowing the agents to maintain beliefs up to only *a finite level of this recursive specification.* An agent's *strategy level* is then defined based on how many levels of recursive beliefs it is able to model and maintain as part of its specification. Agents that maintain higher orders of beliefs are considered to have a higher strategy level than agents

that do not. From an epistemological standpoint, I use this model to illustrate deep levels of Bayesian belief update that take place in the reasoning process of a rational agent in an interactive setting. From a computational and algorithmic standpoint, I study the running time of this model and characterize its computational complexity. Further, I develop a *memoized dynamic programming* alogirithm that deals with the dimensionality of this problem in a flexible way by achieving a trade-off between optimal (fine-tuned) behavior and speed; thereby, providing a decision-theoretic and complexity-theoretic realization of *bounded rationality*. This work also is yet to be communicated.

Thirdly, I am searching for a characterization of the relationship between the two approaches – i.e. game-theoretic equilibria in incomplete information settings and decision-theoretic (subjective expected) optimal strategies. In particular, I am curious about epistemic conditions [4] under which the strategies of decision-theoretic agents resemble or even converge to game-theoretic equilibrium behavior ([14] and [15]). I intend to further explore these ideas and, as in prior work, will continue to use bilateral bargaining under uncertainty as my main object of study.

And, lastly, I am interested in examining the applicability and extensibility of Dirichlet process priors ([16] and [1]), which have been successfully used to represent higher order probability distributions for nonparametric statistical inference problems, to Bayesian belief update problems for (interactive) decision processes.

2 Other Research

A recently begun research project I am involved in deals with searching for policy-improvement type algorithms for mean (Cesaro) payoff stochastic games (along the lines of [23] and [10]).

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