Math 535 Homework 3

Due February 15

1) Find a Möbius transformation T taking the circle $|z| = 1$ to the circle $|z + 2| = 1$, such that $T(-1) = -3$ and $T(i) = -1$.

2) #2, p. 108 of Ahlfors

3) #2, p. 120 of Ahlfors

4) Suppose γ is a curve parameterized by a function $z(t)$ on $[a, b]$, and suppose $f(z)$ is an analytic function on the image of $z(t)$. Define $f(\gamma)$ to be the curve parameterized by $f(z(t))$ on $[a, b]$. Show that for any continuous function $g(z)$ on a neighborhood of the image of $f(z(t))$, one has

$$\int_\gamma g(f(z))f'(z)\,dz = \int_{f(\gamma)} g(z)\,dz$$

5) Suppose $f(z)$ is analytic on a (connected) region Ω such that $Re(f(z)) > 0$ for all z. Show that for every closed curve γ in Ω the following holds.

$$\int_\gamma \frac{f'(z)}{f(z)}\,dz = 0$$

6) Suppose $f(z)$ is analytic on (the image of) a closed curve γ, such that $|f(z)| = 1$ on γ. Show that the following quantity is an integer multiple of $2\pi i$:

$$\int_\gamma \bar{f}(z)f'(z)\,dz$$