

CS 401: Computer Algorithms I

Stable Matching

Xiaorui Sun

This Course

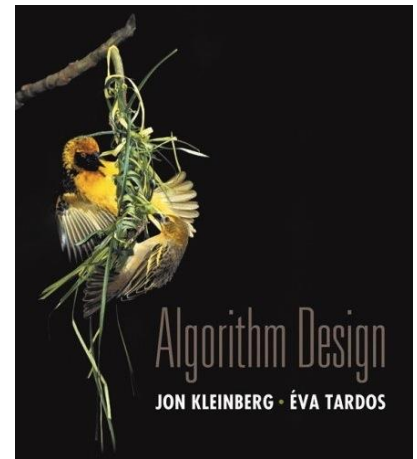
Talk about how to solve problems using computers, aka, algorithms.

Goal:

- Learn algorithm design principles
- How to analyze the runtime
- Correctness (will be discussed in class, but formal proofs are not required by homework and exams)
- Understand some problems are difficult

Content: Most of chapter 1-8

- Representative problems
- Basic of algorithm analysis
- Graphs
- Greedy algorithms
- Divide and conquer
- Dynamic programming
- Max flow (optional)
- NP and Computational Intractability



Course textbook

This Course

Talk about how to solve problems using computers, aka, algorithms.

Goal:

- Learn algorithm design principles
- How to analyze the runtime
- Correctness (will be discussed in class, but formal proofs are not required by homework and exams)
- Understand some problems are difficult

Prerequisite: (CS 151 and CS 251)

- Discrete math (set, tree, graph)
- Data structures (queue, stack, tree, graph)
- Basic proof techniques (contradiction, induction)

This Course

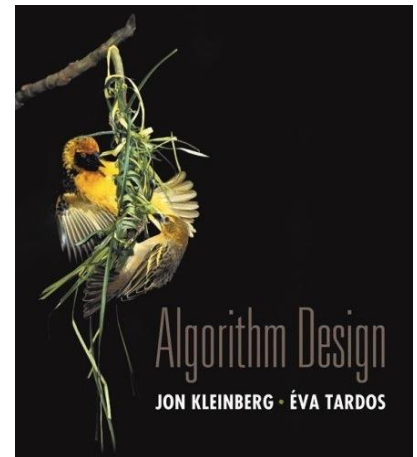
Talk about how to solve problems using computers, aka, algorithms.

Goal:

- Learn algorithm design principles
- How to analyze the runtime
- Correctness
- Understand some problems are difficult

Grading Scheme

- Homework ~ 45%
2 writing + 3 programming
- Midterm ~ 20%
- Final ~ 30%
- Participation ~ 5%
~13 in-class quizzes via pollev.com
10 participations receive full credits

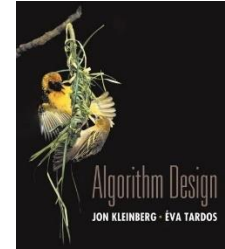


Course textbook

TA: Dong Cao, Chunyu Miao

Where to get help?

- Ask question in the class
- Read the textbook!
- Website: <http://www.cs.uic.edu/~xiaorui/cs401>
 - Lecture slides, homework
- Piazza:
<https://piazza.com/uic/spring2025/cs40141266412674126841269>
 - Announcements, online discussion forum
 - TA will answer course related questions
- Blackboard
 - Office hour, lecture video recordings, homework submission
- Lectures will be video recorded, but no live streaming
- Office hours
- Myself: Thursday 11am-1pm SEO 1241 or blackboard
- TAs: TBA



Stable Matching Problem

Given n men and n women, find a “stable matching”.

- We know the preference of all people.

	favorite	least favorite	
	1 st	2 nd	3 rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

Men's Preference Profile

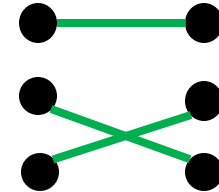
	favorite	least favorite	
	1 st	2 nd	3 rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

Stable Matching

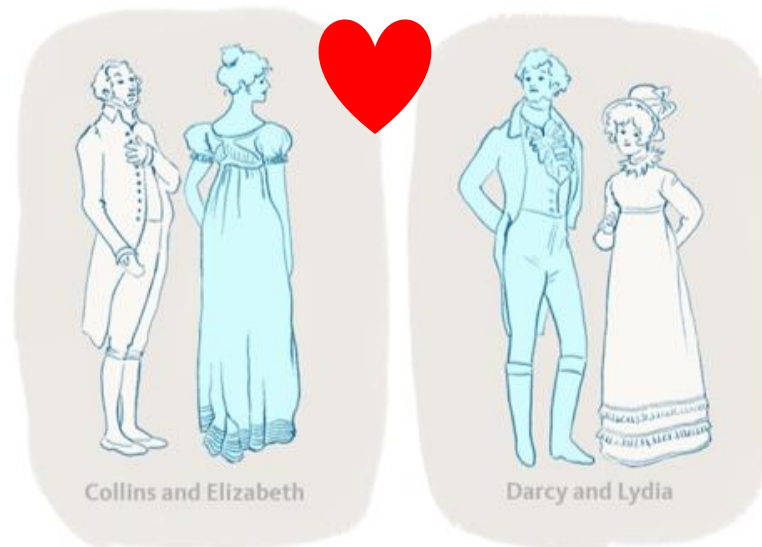
Perfect matching:

- Each man gets exactly one woman.
- Each woman gets exactly one man.



Stability: no incentive to exchange

- an unmatched pair **m-w** is **unstable**
- if man **m** and woman **w** prefer each other to current partners.



Collins and Elizabeth

Darcy and Lydia

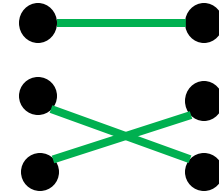
An unstable match:

ELIZABETH AND DARCY LIKE EACH OTHER BETTER THAN THEIR PARTNERS

Stable Matching

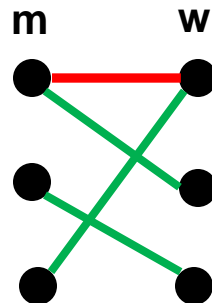
Perfect matching:

- Each man gets exactly one woman.
- Each woman gets exactly one man.



Stability: no incentive to exchange

- an unmatched pair **m-w** is **unstable**
- if man **m** and woman **w** prefer each other to current partners.



Stable matching: perfect matching with no unstable pairs.

Stable matching problem: Given the preference lists of **n** men and **n** women, find a stable matching if one exists.

Example

Question. Is assignment X-C, Y-B, Z-A stable?

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

Men's Preference Profile

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

Example

Question. Is assignment **X-C**, **Y-B**, **Z-A** stable?

Answer. No. Brenda and Xavier will hook up.

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

Men's Preference Profile

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

Example (cont'd)

Question: Is assignment X-A, Y-B, Z-C stable?

Answer: Yes.

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

Men's Preference Profile

	favorite ↓		least favorite ↓
	1 st	2 nd	3 rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

Existence of Stable Matchings

Question. Do stable matchings always exist?

Answer. Yes, but not obvious.

Stable roommate problem: (or “same-sex” stable matching)

2n people; each person ranks others from **1** to **2n-1**.

Assign roommate pairs so that no unstable pairs.

	<i>1st</i>	<i>2nd</i>	<i>3rd</i>
<i>Adam</i>	B	C	D
<i>Bob</i>	C	A	D
<i>Chris</i>	A	B	D
<i>David</i>	A	B	C

A-B, C-D \Rightarrow B-C unstable
A-C, B-D \Rightarrow A-B unstable
A-D, B-C \Rightarrow A-C unstable

So, Stable matchings do not always exist for stable roommate problem.

Propose-And-Reject Algorithm [Gale-Shapley'62]

```
Initialize each person to be free.
while (some man is free and hasn't proposed to every woman) {
    Choose such a man m
    w = 1st woman on m's list to whom m has not yet proposed
    if (w is free)
        assign m and w to be engaged
    else if (w prefers m to her current partner m')
        assign m and w to be engaged, and m' to be free
    else
        w rejects m
}
```

Theorem: Propose-And-Reject Algorithm always finds a stable matching.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Victor proposes to Brenda.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Victor proposes to Brenda.

- Brenda accepts since previously unmatched.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Wyatt proposes to Diane.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Wyatt proposes to Diane.

- Diane accepts since previously unmatched.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile


	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Xavier proposes to Brenda.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor


Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire


Xavier proposes to Brenda.

- Brenda dumps Victor and accepts Xavier.

Women's Preference Profile


	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile


	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Victor proposes to Amy.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor


Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire


Victor proposes to Amy.

- Amy accepts since previously unmatched.

Women's Preference Profile


	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile



	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Yuri proposes to Amy.



Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri		Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire



Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt		Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor



Yuri proposes to Amy.

- Amy rejects since she prefers Victor.

Men's Preference Profile




	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri		Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile




	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt		Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Yuri proposes to Diane.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt		Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri		Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire




Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt		Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	
Erika	Yuri	Wyatt	Zoran	Xavier	Victor




Yuri proposes to Diane.

- Diane dumps Wyatt and accepts Yuri.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor		Amy	Diane	Erika	Claire
Wyatt		Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri		Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt		Xavier
Brenda	Xavier	Wyatt	Yuri		Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Wyatt proposes to Brenda.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Brenda	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Zoran	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Zoran
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Wyatt proposes to Brenda.
 - Brenda rejects since she prefers Xavier.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Zoran	Xavier
Brenda	Xavier	Wyatt	Yuri	Brenda	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Erika
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Wyatt proposes to Amy.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Wyatt proposes to Amy.

- Amy rejects since she prefers Victor.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Zoran	Victor
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Brenda	Erika	Xavier
Brenda	Xavier	Wyatt	Yuri	Diane	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Brenda
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Wyatt proposes to Claire.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Zoran	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Wyatt proposes to Claire.

- Claire accepts since previously unmatched.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Zoran	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Zoran proposes to Brenda.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Zoran proposes to Brenda.

- Brenda rejects since she prefers Xavier.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Zoran proposes to Diane.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Zoran proposes to Diane.

- Diane rejects Yuri and accepts Zoran.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Wyatt	Xavier	Yuri
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Claire	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Wyatt	Xavier	Yuri
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Yuri proposes to Claire.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Erika	Claire
Zoran	Brenda	Diane	Amy	Erika	Claire

Yuri proposes to Claire.

- Claire rejects since she prefers Wyatt.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Zoran	Zoran	Victor
Diane	Victor	Zoran	Wyatt	Xavier	Yuri
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Brenda	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Brenda	Erika	Xavier
Brenda	Xavier	Wyatt	Yuri	Victor	Zoran
Claire	Wyatt	Xavier	Zoran	Zoran	Victor
Diane	Victor	Zoran	Erika	Xavier	Brenda
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Yuri proposes to Brenda.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Erika	Claire
Zoran	Brenda	Diane	Amy	Erika	Claire

Yuri proposes to Brenda.

- Brenda rejects since she prefers Xavier.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Victor	Erika	Zoran
Claire	Wyatt	Xavier	Zoran	Victor	Erika
Diane	Victor	Zoran	Wyatt	Erika	Xavier
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Claire	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Brenda	Claire	Xavier
Brenda	Xavier	Wyatt	Victor	Erika	Zoran
Claire	Wyatt	Xavier	Zoran	Zoran	Victor
Diane	Victor	Zoran	Claire	Xavier	Brenda
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Yuri proposes to Erika.

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Claire	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Yuri proposes to Erika.

- Erika accepts since previously unmatched.

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Yuri	Xavier
Brenda	Xavier	Wyatt	Victor	Yuri	Zoran
Claire	Wyatt	Xavier	Yuri	Zoran	Victor
Diane	Victor	Zoran	Yuri	Xavier	Wyatt
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

Men's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Victor	Brenda	Amy	Diane	Erika	Claire
Wyatt	Diane	Brenda	Amy	Claire	Erika
Xavier	Brenda	Erika	Claire	Diane	Amy
Yuri	Amy	Diane	Brenda	Claire	Erika
Zoran	Brenda	Diane	Amy	Erika	Claire

Women's Preference Profile

	0 th	1 st	2 nd	3 rd	4 th
Amy	Zoran	Victor	Wyatt	Xavier	
Brenda	Xavier	Wyatt	Victor	Zoran	
Claire	Wyatt	Xavier	Zoran		Victor
Diane	Victor	Zoran	Xavier		
Erika	Yuri	Wyatt	Zoran	Xavier	Victor

STOP

- Everyone matched.
- Stable matching!

Questions

- Q: Why GS algorithm solves Stable Matching problem?
- Q: How to implement GS algorithm efficiently?

What do we need to prove?

Goal: prove Propose-And-Reject Algorithm always finds a stable matching.

- The algorithm ends.
- The output is correct.

Correctness proof is not required in the homework and exams.

But understanding the correctness helps you develop correct algorithms.

Propose-And-Reject Algorithm [Gale-Shapley'62]

Initialize each person to be free.

```
while (some man is free and hasn't proposed to every woman) {  
    Choose such a man  $m$   
     $w = 1^{\text{st}}$  woman on  $m$ 's list to whom  $m$  has not yet proposed  
    if ( $w$  is free)  
        assign  $m$  and  $w$  to be engaged  
    else if ( $w$  prefers  $m$  to her fiancé  $m'$ )  
        assign  $m$  and  $w$  to be engaged, and  $m'$  to be free  
    else  
         $w$  rejects  $m$   
}
```

What do we need to prove?

Goal: prove Propose-And-Reject Algorithm always finds a stable matching.

- The algorithm ends.
How many iterations it takes?
- The output is correct.
It find a **perfect** matching that is **stable**.