Understanding the Solution

Q. For a given problem instance, there may be several stable matchings. Do all executions of Gale-Shapley yield the same stable matching? If so, which one?

An instance with two stable matchings:

- A-X, B-Y.
- A-Y, B-X.

	1st	2nd
Xavier	A	B
Yuri	B	A

	1st	2nd
Amy	Y	X
Brenda	X	Y

Man Optimal Assignments

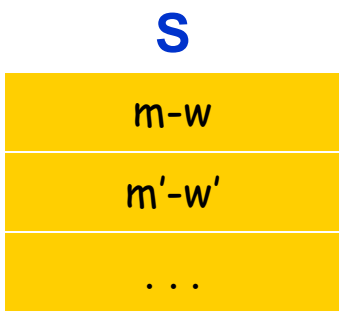
Definition: Man **m** is a valid partner of woman **w** if there exists some stable matching in which they are matched.

Man-optimal assignment: Each man receives the best valid partner (according to his preferences).

- Is man-optimal assignment a matching?
- Simultaneously best for each and every man.

Claim: All executions of GS yield a man-optimal matching, which is a stable matching!

Man Optimality



Claim: GS matching **S*** is man-optimal.

Proof: (by contradiction)

Suppose some man is paired with someone other than his best partner. Men propose in decreasing order of preference ⇒ some man is rejected by a valid partner.

Let **m** be the man who is the first such rejection, and let **w** be the women who is first valid partner that rejects him.

Let **S** be a stable matching where **w** and **m** are matched.

In building **S***, when **m** is rejected, **w** forms (or reaffirms) engagement with a man, say **m'**, whom she prefers to **m**.

Let **w'** be the partner of **m'** in **S**.

In building **S***, **m'** is not rejected by any valid partner at the point when **m** is rejected by **w**. Thus, **m'** prefers **w** to **w'**.

since this is the **first** rejection by a valid partner

But **w** prefers **m'** to **m**.

Thus **w-m'** is unstable in **S**.

■

Man Optimality Summary

Man-optimality: In version of GS where men propose, each man receives the best valid partner.

w is a valid partner of **m** if there exist some stable matching where **m** and **w** are paired