

Homework 2: Due May 9, 2022

Please answer at least **three** of the following questions.

Problem 1:

What is the most interesting representation learned **from this course** to you, and why?

Problem 2:

What is the most interesting representation **you think that should be introduced in this class but did not**, and why?

Problem 3:

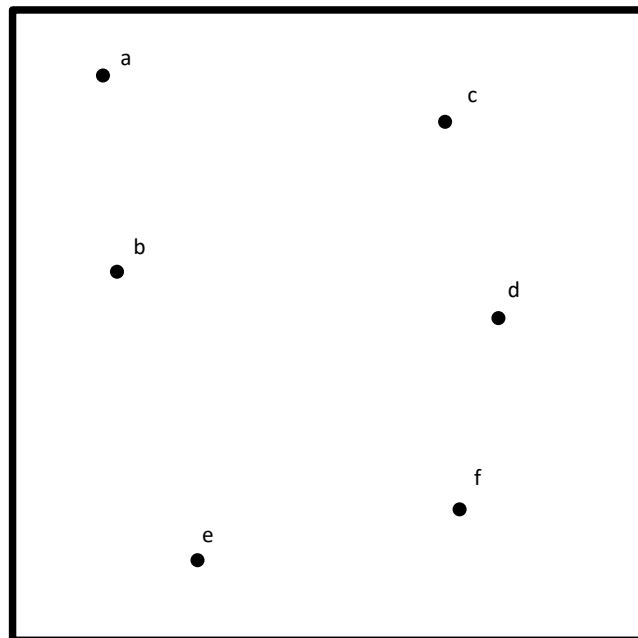
Show that $R(\langle 2, 2, 2 \rangle) \leq 7$ (the rank of matrix multiplication tensor $\langle 2, 2, 2 \rangle$ is smaller than or equal to 7) by writing $\langle 2, 2, 2 \rangle$ as a linear combination of rank 1 tensors.

Problem 4:

Suppose there is a matrix multiplication tensor $\langle K, N, W \rangle$ such that $R(\langle K, N, W \rangle) \leq r$. Describe a matrix multiplication algorithm with running time $n^{3 \log r / \log(KNM)}$ using this tensor.

Problem 5:

Give a 2-WSPD of the following points in a 2-dimensional space.



Problem 6:

Show that the $(1 + \epsilon)$ -spanner construction based on BSPD is a connected graph. Recall that the spanner is constructed as follows:

1. Find a $1/(1 + \delta)$ -WSPD, where $\delta = \epsilon/32$.
2. For each pair (A_i, B_i) in the WSPD, choose an arbitrary $x \in A_i$ and an arbitrary $y \in B_i$, add edge (x, y) to the spanner with edge weight $d(x, y)$.