$\mathcal{T}$: - foreach tree $T \in \mathcal{T}$

$$d_T(u,v) \geq d_G(u,v)$$

$$- \mathbb{E}_{T \in \mathcal{T}}[d_T(u,v)] \leq O(\log n \cdot \log \Delta) \, d_G(u,v).$$

Bartal's Algo [Bartal 1996].

$\rightarrow$ Based on New LDD.

LDD Algo (G with ~~diameter~~ parameter Δ)

- R randomly sampled from $[\frac{\Delta}{4}, \frac{\Delta}{2}]$.
- Permutation (random) on V. $\leftarrow \pi$
- $c(v) \leftarrow$ NULL.
- for each vertex $i = 1$ to n.

  Look at vertices v s.t. ① $c(v) = $ NULL

  and ② $d_G(\pi(i), v) \le R$.

  $c(v) = i$

- $\{V_i = \{v : c(v) = i\}\}$.

within each class

Dist $(u, v) \le \Delta$

Lemma: For each edge $e=(u,v)$

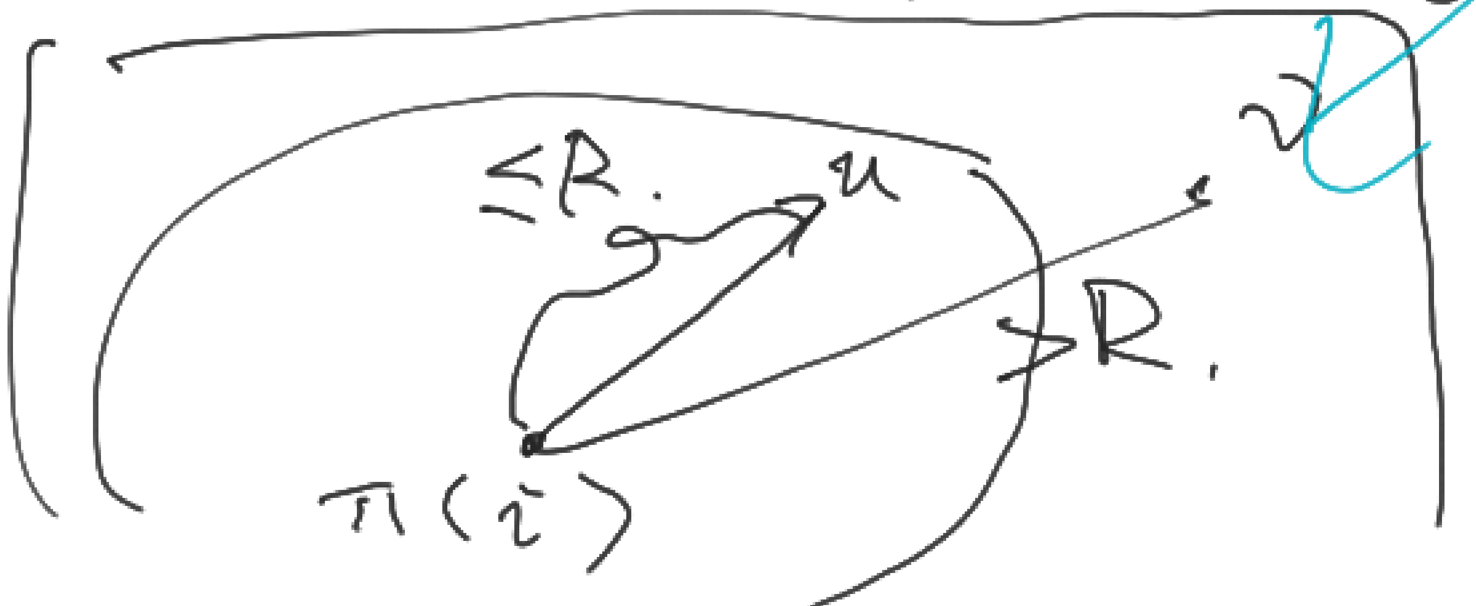
$$\Pr[\underset{\underset{(u,v)}{\parallel}}{e} \text{ is an crossing edge}] \le \frac{d_G(v,u)\cdot \log n}{\Delta}$$

when e is an crossing edge.

$\hookrightarrow$ $\underbrace{c(u)}_{\underset{i}{\parallel}} \neq \underbrace{c(v)}_{\underset{j}{\parallel}}$.

$i < j$

i-th vertex cutted edge e.



observation 1: if e is an crossing edge.

$dist_G(\pi(i), u) \le R$.

$dist_G(\pi(i), v) > R$.

Claim: If edge e is cutted by vertex x, then $R \in [\mathrm{dist}_G(x,u), \mathrm{dist}_G(x,v)]$.

Pf: Fix $e=(u,v)$

for another vertex x.

- $L_x = \min\{\mathrm{dist}_G(x,u), \mathrm{dist}_G(x,v)\}$
- $U_x = L_x + \mathrm{dist}_G(u,v)$

$\rightarrow R \in [L_x, U_x)$ $\leftarrow$ one necessary condition fix $e = (u, v)$.

$\frac{\Delta}{4}$ $\frac{\Delta}{2}$,

when x_j cut edge e? (for π).

$< x_i \quad x_k < y_j \; x_j \qquad (k<j).$

rank of x_k < rank of x_j.

$\hookrightarrow$ x_j cannot cut edge e.

one of $u, v \in B(x_k, R)$.

necessary condition for x cut e.

in π, $\pi^{-1}(x_k) \geq \pi^{-1}(x_j)$

for all $k < j$.

Observation: x_j cut edge e.

① $R \in [L_{x_j}, U_{x_j}]$.

② ~~$\forall$~~ for all $k < j$, $\pi^{-1}(x_k) > \pi^{-1}(x_j)$.

$\Pr[e \text{ is cutted by } ~~edge~~ \; x_j] \le ?$

$\Pr[R \in [L_{x_j}, U_{x_j}]] \le \frac{d(u,v)}{\Delta/4}$.

$\Pr[② \text{ condition hold}] \le \frac{1}{j}$

if x_j is rank first among $x_1 \dots x_j$ then probably x_j cut e

$\Pr[e$ is cutted by $x_j]$.

$= \Pr[R \in [L_{x_j}, U_{x_j}]]$ and

x_j is ranked first among $x_1 \ldots x_j]$

$= \Pr[R \in [\quad]] \cdot \Pr[x_j$ is first$]$

$$\leq \frac{d(u,v)}{\frac{\delta}{4}} \cdot \frac{1}{j} = \frac{4d(u,v)}{j}.$$

Pr[e is cutted by some vertex]

$$= \sum_{x_j} \Pr[e \text{ is cutted by } x_j].$$

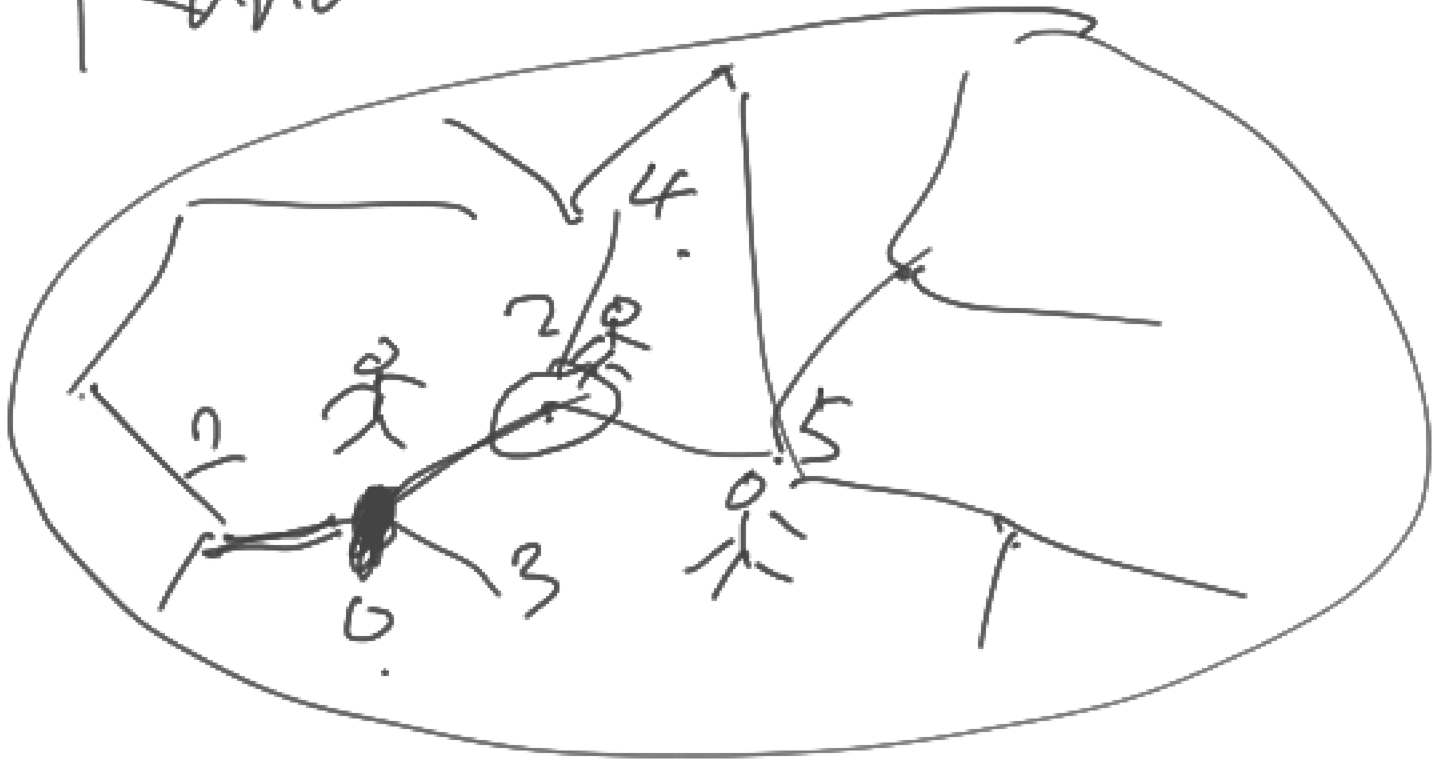
$$\le \sum_{j=1}^{n} \frac{4d(u,v)}{\Delta \cdot j} \le \underbrace{\frac{4 \cdot d(u,v) \cdot \log n}{\Delta}}$$

$$\sum_{j=1}^{n} \frac{1}{j} = \log n$$

G is "well connected".

$\Rightarrow$ [In graph G, random walk from vertex s to vertex t is $O(n)$. [expectation]].

Random walk.



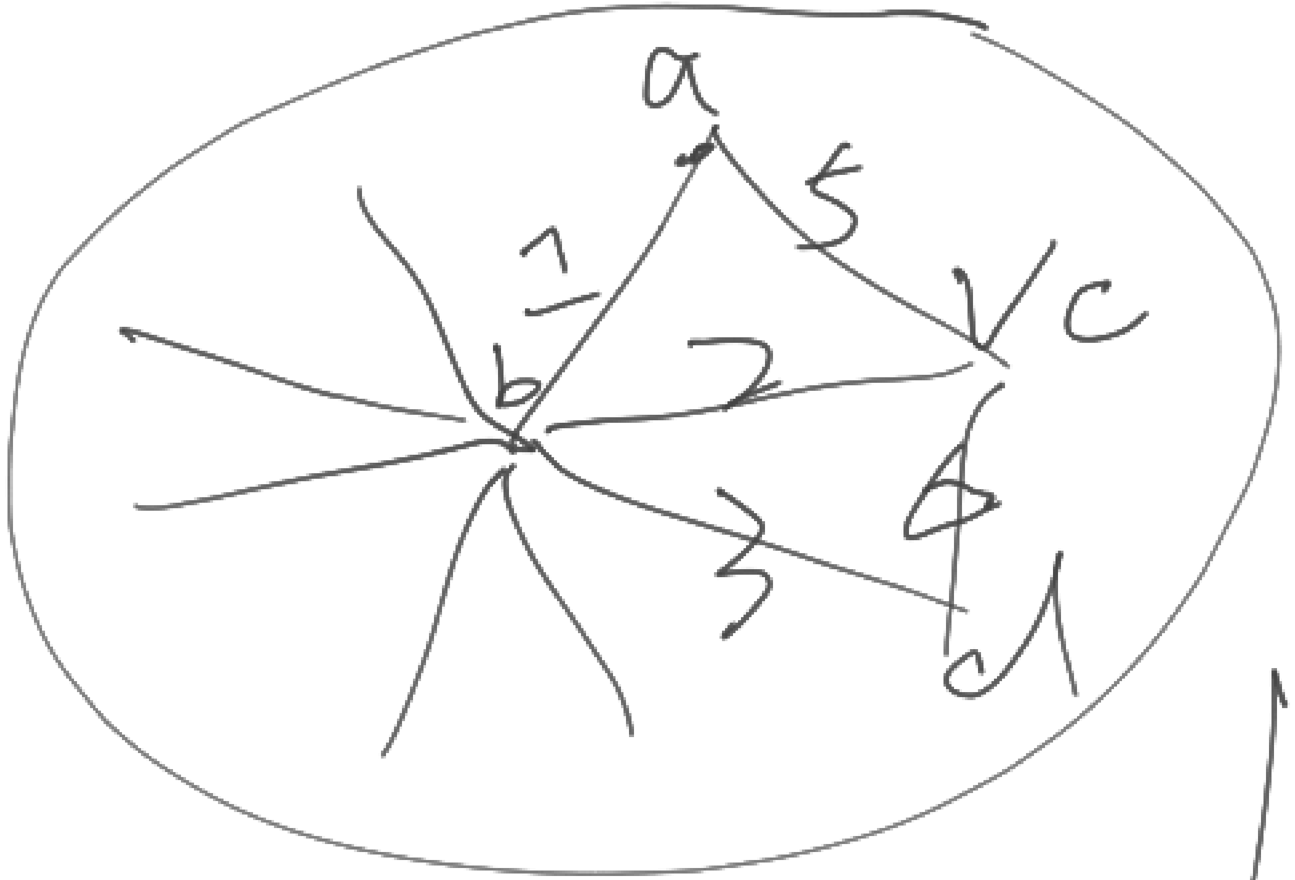
person at vertex 1 with probability $\frac{1}{3}$ in next time step.

2

$\frac{1}{3}$

3

$\frac{1}{3}$



random walk from a in one step.

with probability $\frac{1}{1+5} = \frac{1}{6}$.

move to b

— — — — — $\frac{5}{1+5} = \frac{5}{6}$

move to c.

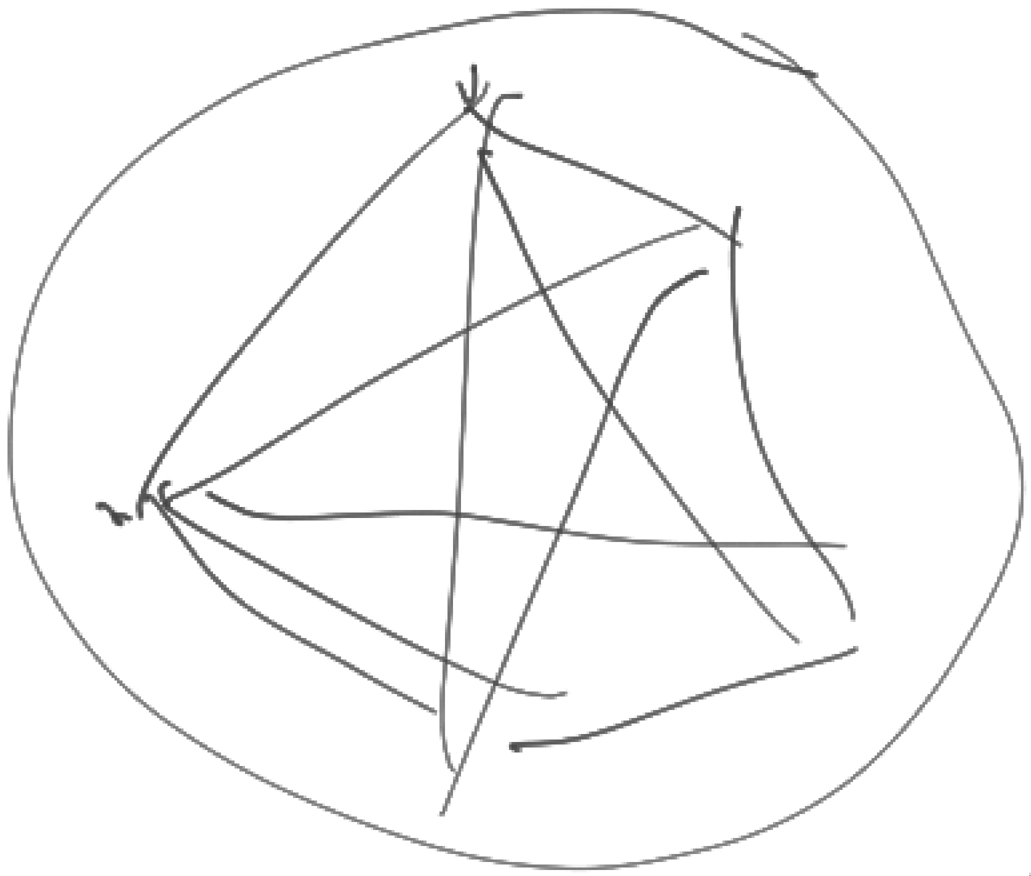
with probility $\frac{5}{5+2+6} = \frac{5}{13}$

move to a

$\frac{2}{13} \longrightarrow b$

$\frac{6}{13} \longrightarrow d.$

Graph is a complete graph.

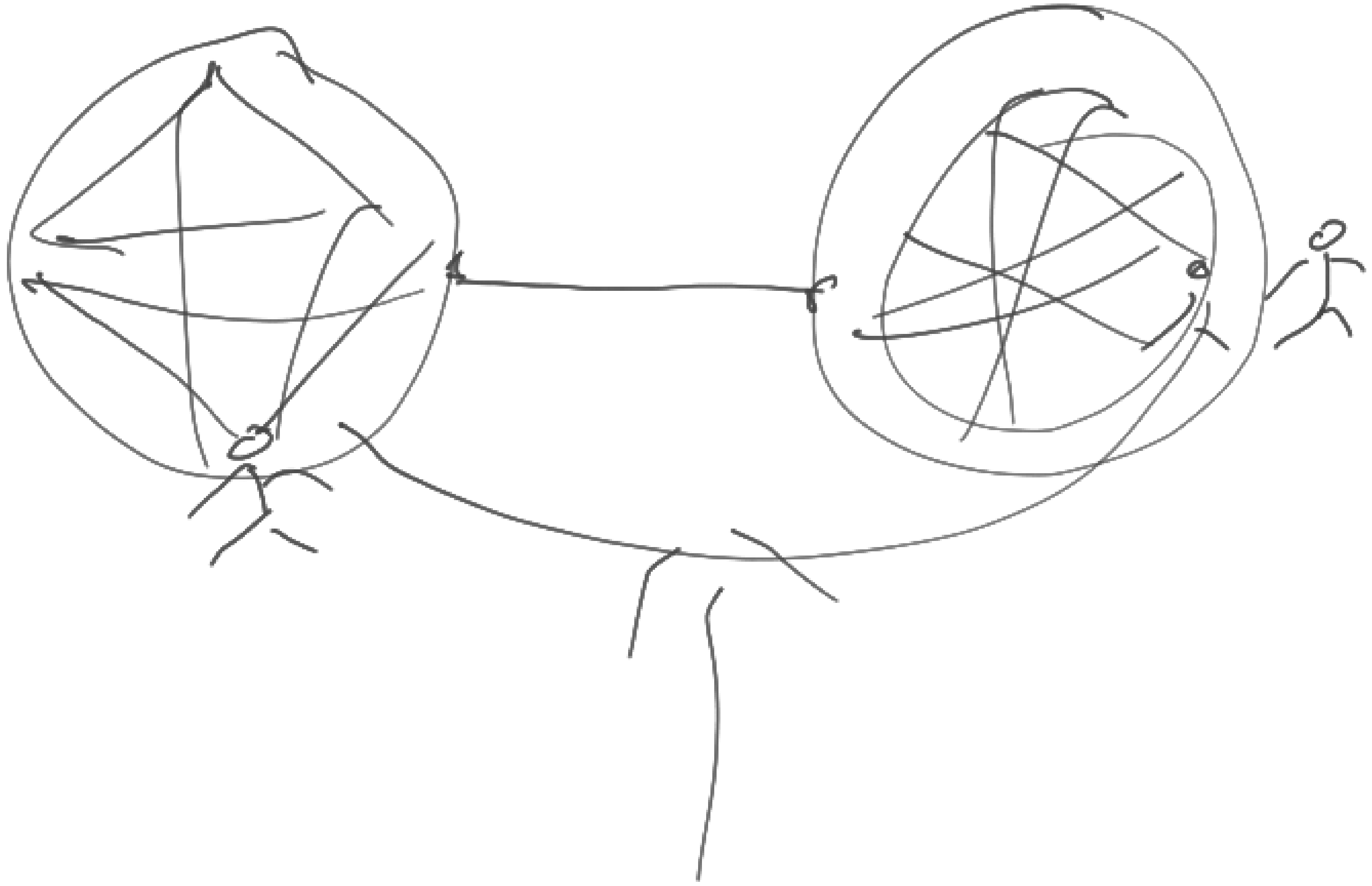


Claim: complete graph is a well connected graph.

After ~~one~~ any step walk.

$\Pr[\text{at vertex } v] = \frac{1}{n}$.

HW.



not a well connected graph.