for $v = 1$ to $n-1$ do
  for $r = v+1$ to n do
    for $c = v+1$ to n do

$$A[r,c] = -\frac{A[v,c]}{A[v,v]} * A[r,v]$$

$$b[r] = -\frac{b[v]}{A[v,v]} * A[r,v]$$

$$A[r,v] = 0$$

$$a_{00}x_0 + a_{01}x_1 + a_{02}x_2 = b_0$$

$$a_{10}x_0 \quad a_{11}x_1 \quad a_{12}x_2 = b_1$$

$$a_{20}x_0 \quad a_{21}x_1 \quad a_{22}x_2 = b_2$$

$$x_0 = \frac{b_0}{a_{00}} - \frac{a_{01}}{a_{00}}x_1 - \frac{a_{02}}{a_{00}}x_2$$

$$a_{10}\left[\frac{b_0}{a_{00}} - \frac{a_{01}}{a_{00}}x_1 - \frac{a_{02}}{a_{00}}x_2\right] + a_{11}x_1 + a_{12}x_2 = b_1$$

$$x_1\left[a_{11} - a_{10} \times \frac{a_{01}}{a_{00}}\right] + x_2\left[a_{12} - a_{10} \times \frac{a_{02}}{a_{00}}\right] = b_1 - a_{10} \times \frac{b_0}{a_{00}}$$

$$x_1\left[a_{21} - a_{20} \times \frac{a_{01}}{a_{00}}\right] + x_2\left[a_{22} - a_{20} \times \frac{a_{02}}{a_{00}}\right] = b_2 - a_{20} \times \frac{b_0}{a_{00}}$$

Transform to unit upper diagonal matrix A'.

$$
\overset{A}{\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}} \overset{=}{=} \overset{L}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ L_{21} & 1 & 0 & 0 \\ L_{31} & L_{32} & 1 & 0 \\ L_{41} & L_{42} & L_{43} & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix}}
$$

$$
A = \begin{bmatrix} \cancel{L_{11}}U_{11} & \cancel{L_{11}}U_{12} & \cancel{L_{11}}U_{13} & \cancel{L_{11}}U_{14} \\ L_{21}U_{11} & L_{21}U_{12} + \cancel{L_{22}}U_{22} & L_{21}U_{13} + \cancel{L_{22}}U_{23} & L_{21}U_{14} + \cancel{L_{22}}U_{24} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + \cancel{L_{33}}U_{33} & L_{31}U_{14} + L_{32}U_{24} + \cancel{L_{33}}U_{34} \\ L_{41}U_{11} & L_{41}U_{12} + L_{42}U_{22} & L_{41}U_{13} + L_{42}U_{23} + L_{43}U_{33} & L_{41}U_{14} + L_{42}U_{24} + L_{43}U_{34} + \cancel{L_{44}}U_{44} \end{bmatrix}
$$

```
for k = 1 to n do
    for j = k+1 to n do
        A[j,k] ← A[j,k] / A[k,k]
    for j = k+1 to n do
        for i = k+1 to n do
            A[i,j] ← A[i,j] − A[i,k] * A[k,j]
```

http://en.wikipedia.org/wiki/LU-decomposition

(1) $Ax = b = \underbrace{LU}\, x = b$

Solve $Ly = b$, then $Ux = y$

Computationally efficient when solving matrix eqn multiple times for diff $\vec{b}$.

(2) Determinant: $\det(A) = \det(L) \cdot \det(U) = 1 \times \prod_{1}^{n} u_{ii}$

0/1 - Knapsack

Input : n objects, & $\begin{matrix} v_i \\ w_i \end{matrix}$ per object ; W

Goal : maximize $\sum_{i=1}^{n} b_i v_i$, $b_i = 0, 1$

subject to $\sum_{i=1}^{n} b_i w_i \leq W$

<u>Option 1</u>: Exhaustive search of 2^n combos. $\Theta(2^n)$

<u>Option 2</u>: Dynamic programming using table, cost $\Theta(nW)$

// pseudo-polynomial, ie, polynomial in VALUE of input W

$A(0, W) = 0$

$A(i, 0) = 0$

$$A(i, W) = A(i-1, W) \text{ if } w_i \not\leq W$$

$$= \max\left[v_i + A(i-1, W - w_i),\ A(i-1, W)\right] \text{ if } w_i \leq W$$

e.g. $n = 3$

w_i	3	6	8
v_i	4	7	9

i \ W	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	4	4	4	4	4	4	4
2	0	0	0	4	4	4	7	7	7	11
3	0	0	0	4	4	4	7	7	9	11