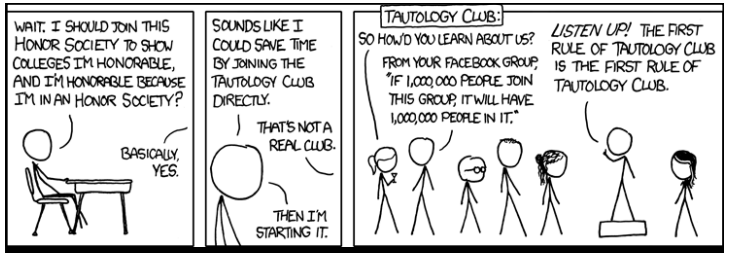


Randall Munroe, XKCD: http://xkcd.com/703/

CS151 Fall 2014
Lecture 3 – 9/2
Propositional Logic

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/CS151

Adapted from Lap Chi Lau and David Liben-Nowell

Announcements

- Friday, September 5th is the last day for add/drop for Fall '14
- Prerequisites are taken seriously.
  Prerequisite(s): MATH 180; and Grade of C or better in CS 111
- Homework 1 is out. Due in 1 week (Sept 9) in class
- Engineering Resume Expo
  When: Thursday, September 18th, 2014
  Time: 8:45 AM- 12:00 PM
  Where: Student Center East Tower- Room 302

2

Announcements

First ACM Meeting

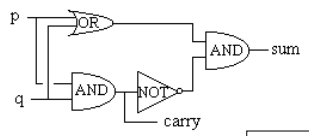
When: Thursday, September 4th, 2014 at 5:30pm
Where: CS Lounge (SELE 2260)
Why: Come hear about what's new, get involved, and pizza!

ACM (Association for Computing Machinery)
is a student organization for anything with computers and technology. We host meetings on career development, technical talks, and professor research. We have SIGs (special interest groups) that focus on different topics and projects like open-source and app development. Come to the meeting to find out about what ACM is, what's going on this year, and just to say hi!

3

Writing Logical Formula for a Truth Table

Digital logic:



p	q	sum	carry
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	0

	p	q	r	output
$p \wedge q \wedge r$	T	T	T	F
$p \wedge q \wedge \neg r$	T	T	F	T
$p \wedge \neg q \wedge r$	T	F	T	T
$p \wedge \neg q \wedge \neg r$	T	F	F	F
$\neg p \wedge q \wedge r$	F	T	T	T
$\neg p \wedge q \wedge \neg r$	F	T	F	T
$\neg p \wedge \neg q \wedge r$	F	F	T	T
$\neg p \wedge \neg q \wedge \neg r$	F	F	F	F

Idea 2: Look at the false rows, negate and take the "and".

$$\neg(p \wedge q \wedge r)$$
$$\wedge\neg(p \wedge \neg q \wedge \neg r)$$
$$\wedge\neg(\neg p \wedge \neg q \wedge \neg r)$$

can be simplified further

The formula is true iff the input is not one of the false row.

DeMorgan's Laws

Logical equivalence: Two statements have the same truth table

De Morgan's Law $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

De Morgan's Law $\neg(p \vee q) \equiv \neg p \wedge \neg q$

DeMorgan's Laws

Logical equivalence: Two statements have the same truth table

De Morgan's Law $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Statement: Tom is in the football team and the basketball team.
Negation: Tom is not in the football team or not in the basketball team.

De Morgan's Law $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Statement: The number 783477841 is divisible by 7 or 11.
Negation: The number 783477841 is not divisible by 7 and not divisible by 11.

Distributive laws

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

For more examples, see Rosen Table 6, page 27

Simplifying Statement

$$\begin{aligned}
&\neg(\neg p \wedge q) \wedge (p \vee q) \\
&\equiv (\neg\neg p \vee \neg q) \wedge (p \vee q) && \text{DeMorgan} \\
&\equiv (p \vee \neg q) \wedge (p \vee q) \\
&\equiv p \vee (\neg q \wedge q) && \text{Distributive} \\
&\equiv p \vee \text{False} \\
&\equiv p
\end{aligned}$$

Conditional Statement

If p then q $p \longrightarrow q$

p is called the hypothesis; q is called the conclusion

The department says: "If your GPA is 4.0, then you don't need to pay tuition fee."

When is the above sentence false?

- It is false when your GPA is 4.0 but you still have to pay tuition fee.
- But it is not false if your GPA is below 4.0.

Another example: "If there is typhoon T8 today, then there is no class."

When is the above sentence false?

Logic Operator

$\longrightarrow$ ::= IMPLIES

P	Q	$P \longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

False implies anything!

Convention: if we don't say anything wrong, then it is not false, and thus true.

If-Then as Or

$p \longrightarrow q \equiv ?$

P	Q	$P \longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Idea 2: Look at the false rows, negate and take the "and".

$$\neg(P \wedge \neg Q)$$
$$\equiv \neg P \vee Q$$

- If you don't give me all your money, then I will kill you.
- Either you give me all your money or I will kill you (or both).
- If you talk to her, then you can never talk to me.
- Either you don't talk to her or you can never talk to me (or both).

Negation of If-Then

$\neg(p \longrightarrow q) \equiv ?$

- If your GPA is 4.0, then you don't need to pay tuition fee.
- Your term GPA is 4.0 and you still need to pay tuition fee.
- If my computer is not working, then I cannot finish my homework.
- My computer is not working but I can finish my homework.

$$\neg(P \rightarrow Q)$$
$$\equiv \neg(\neg P \vee Q) \quad \text{previous slide}$$
$$\equiv \neg\neg P \wedge \neg Q \quad \text{DeMorgan}$$
$$\equiv P \wedge \neg Q$$

Converse, Contrapositive, Inverse

Statement	$p \rightarrow q$
Converse	$q \rightarrow p$
Contrapositive	$\neg q \rightarrow \neg p$
Inverse	$\neg p \rightarrow \neg q$

Statement	If you are a CS student, then you take CS 151	If you were a POTUS in 2006, then your name is George
Converse		
Contrapositive		
Inverse		

A conditional statement is logically equivalent to its ??

If and only if, Biconditional

↔ *IFF* ⇔

$$p \longleftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

I'll buy the smart phone if you get good grades

I'll buy the smart phone only if you get good grades