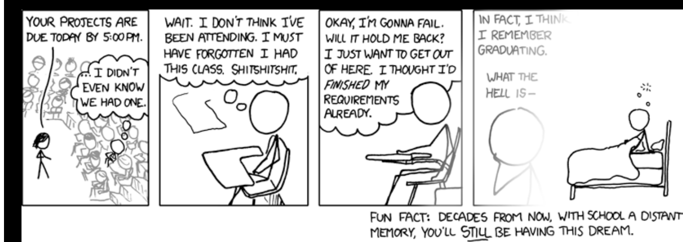
CS151 Fall 2014
Lecture 5 – 9/9

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/bin/view/CS202/WebHome



xkcd comic: **http://xkcd.com/557/**

This event is open to COMPUTER SCIENCE BS and MS students:

It's time….register now to take part in the initial exam for the
ITA Tech Challenge!

- https://app.pitchburner.com/s1/site//itauic
- Thursday, September 11
- Begins at 5:00pm
- SEL 2254

IMPORTANT: Remember to bring a laptop!

Pizza will be provided after the exam in SEL 2260.
Please block off two hours for this program; one hour for the sponsor presentation and one hour for the coding exam.
A representative from CDK Global will make a presentation before the online exam begins.
If you have any questions, please contact Julie Rekar of the Illinois Technology Association at 312-924-1686 or julie@illinoistech.org.

Learn more at itatechchallenge.com and spread the word on Twitter @ITABuzz, #itachallenge.

Contradiction

$$\neg p \rightarrow c$$
$$\therefore\ p$$

If you can show that the assumption that the statement p is false leads logically to a contradiction, then you can conclude that p is true.

C:= You are working as a clerk.
If you have won lottery then you would not work as a clerk
(L -> ~C) AND C
$\therefore$ You have not won lottery. ~L

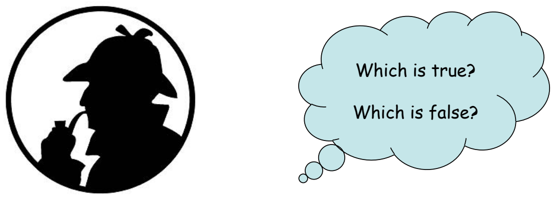
Knights and Knaves

Knights always tell the truth.
Knaves always lie.

A says: B is a knight.
B says: A and I are of opposite type.

Suppose A is a knight.
Then B is a knight (because what A says is true).
Then A is a knave (because what B says is true)
A contradiction.

So A must be a knave.
So B must be a knave (because what A says is false).



"The sentence below is false."

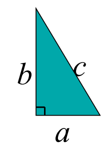
"The sentence above is true."

Limitation of Propositional Logic

Propositional logic – logic of simple statements

$$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$$

How to formulate Pythagoreans' theorem using propositional logic?



How to formulate the statement that there are infinitely many primes?

How to say that a condition holds for all inputs?
Or that there exists a setting of values to variables that satisfy a proposition?

But the fact that some geniuses were laughed at does not imply that all who are laughed at are geniuses. They laughed at Columbus, they laughed at Fulton, they laughed at the Wright brothers. But they also laughed at Bozo the Clown.

---Carl Sagan (1934-1996)

Predicate Logic

Predicates

Predicates are propositions with variables

""The letter **x** is a vowel."

"The integer **x** is prime."

"The string **x** is a palindrome."

"The person **x** costarred in a movie with Kevin Bacon."

"The string **x** is alphabetically after the string **y**."

"The integer **x** evenly divides the integer **y**."

"People **x** and **y** are friends on FB."

"Player **x** will win from TIC-TAC-TOE board position **B** if both players play optimally."

Set

R	Set of all real numbers
Z	Set of all integers
Q	Set of all rational numbers

$x \in A$ means that x is an element of A

$x \notin A$ means that x is not an element of A

Sets can be defined directly:

e.g. {1,2,4,8,16,32,…},

{CS151, CS266, …}

Truth Set

Sets can be defined by a predicate

Given a predicate P(x) and x has domain D, the truth set of P(x) is the set of all elements of D that make P(x) true.

$$\{x \in D \mid P(x)\}$$

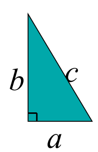
e.g. Let P(x) be "x is the square of an integer", and the domain D of x is set of positive integers.

e.g. Let P(x) be "x is a prime number", and the domain D of x is set of positive integers.

The Universal Quantifier

$\forall x$ For ALL x

∀ x ∈ Z ∀ y ∈ Z, x + y = y + x.



∀ right-angled triangles

$$a^2 + b^2 = c^2$$

The Existential Quantifier

$\exists y$ There EXISTS some y

e.g. $\exists y, y^2 = y$

The truth of a predicate depends on the domain.

$$\forall x \exists y.\ x < y$$

Domain	*Truth value*
integers ☒	T
positive integers ☒$^+$	T
negative integers ☒$^-$	F
negative reals ☒$^-$	T

Translating Mathematical Theorem

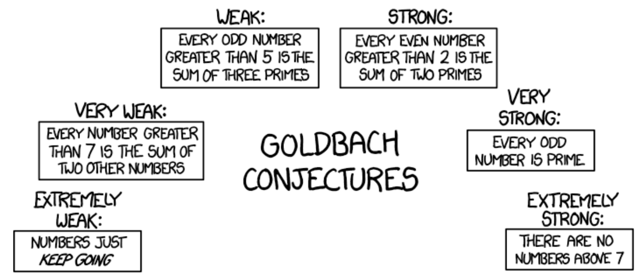
Fermat (1637): If an integer n is greater than 2,

then the equation $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c.

$$\forall n > 2\ \forall a \in Z^+\ \forall b \in Z^+\ \forall c \in Z^+\quad a^n + b^n \neq c^n$$

Andrew Wiles (1994) http://en.wikipedia.org/wiki/Fermat's_last_theorem

Translating Mathematical Theorem



http://xkcd.com/1310/

Translating Mathematical Theorem

Goldbach's conjecture: Every even number is the sum of two prime numbers.

Suppose we have a predicate prime(x) to determine if x is a prime number.

$$\forall n \in Z\ \text{even}(n) \rightarrow$$

$$\exists p \in Z\ \exists q \in Z\ \text{prime}(p)\ \wedge\ \text{prime}(q)\ \wedge\ p+q = n$$

How to write prime(p)?

$$\text{prime}(p)\ :=$$

$$(p > 1) \wedge (\forall a \in Z\ \forall b \in Z\ (a > 1 \wedge b > 1 \rightarrow a \cdot b \neq p))$$