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CS151 Fall 2014
Lecture 6 – 9/11
Propositional Logic

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Adapted from Lap Chi Lau – The Chinese University of Hong Kong
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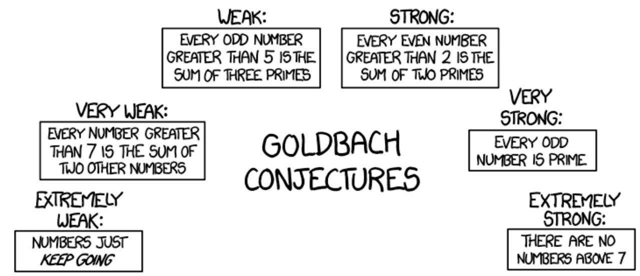
Translating Mathematical Theorem



http://xkcd.com/1310/

$$\forall Even(x), x > 2 \exists Prime(y), Prime(z) \quad x = y + x$$

Programs and predicates
Free and bound variables

```
function (n) {
        for i = 1 to n do
                print i
}
```

P(n) := "function(n) prints integer numbers"

$$\exists n \qquad P(n)$$

Q(n) := "function(n) prints numbers less than 10"

Negations of Quantified Statements

Everyone likes football.

What is the negation of this statement?

Not everyone likes football = There exists someone who doesn't like football.

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

(generalized) DeMorgan's Law

Say the domain has only three values.

$$\neg \forall x P(x) \equiv \neg (P(1) \wedge P(2) \wedge P(3))$$
$$\equiv \neg P(1) \vee \neg P(2) \vee \neg P(3)$$
$$\equiv \exists x \neg P(x)$$

Negations of Quantified Statements

There is a plant that can fly.

What is the negation of this statement?

Not exists a plant that can fly = every plant cannot fly.

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

(generalized) DeMorgan's Law

Say the domain has only three values.

$$\neg \exists x P(x) \equiv \neg(P(1) \vee P(2) \vee P(3))$$
$$\equiv \neg P(1) \wedge \neg P(2) \wedge \neg P(3)$$
$$\equiv \forall x \neg P(x)$$

Order of Quantifiers

There is an anti-virus program killing every computer virus.

How to interpret this sentence?

For every computer virus, there is an anti-virus program that kills it.

$$\forall V \ \exists P, \text{kill}(P, V)$$

http://home.mcafee.com/virusinfo/VirusRemovalTools.aspx

Virus Name	Removal Tool
▸ Sasser	▸ McAfee Stinger
▸ Bagle	▸ McAfee Stinger
▸ Zafi	▸ McAfee Stinger
▸ Mydoom	▸ McAfee Stinger
▸ Lovsan/Blaster	▸ McAfee Stinger
▸ Klez	▸ Klez Removal Tool
▸ Bugbear	▸ Bugbear Removal Tool

"Is Your PC Infected? Don't Worry, We'll Fix It!"

Order of Quantifiers

There is an anti-virus program killing every computer virus.

How to interpret this sentence?

There is one single anti-virus program that kills all computer viruses.

$$\exists P \ \forall V, \text{kill}(P, V)$$

I have *one* defense good against every attack.

Example: P is ??,
protects against *ALL* viruses

That's much better!

Order of quantifiers is very important!

More Negations

There is an anti-virus program killing every computer virus.

$$\exists P \ \forall V, \text{kill}(P, V)$$

What is the negation of this sentence?

$$\neg(\exists P \ \forall V, \text{kill}(P, V))$$
$$\equiv \forall P \ \neg(\forall V, \text{kill}(P, V))$$
$$\equiv \forall P \exists V \neg \text{kill}(P, V))$$

For every program, there is some virus that it can not kill.

Predicate Calculus Validity

Propositional validity

$$p \vee \neg p$$

True ***no matter what*** the truth value of **p** is

Predicate calculus validity

$$\forall x \in S \quad [P(x) \vee \neg P(x)]$$

True ***no matter what***

- the Domain is

  (integers, people, games)
- or the predicates are

  (x > 42, x has red hair, x is similar to Minecraft)

That is, logically correct, independent of the specific content.

Predicate Calculus Validity

Propositional validity

$$(A \rightarrow B) \vee (B \rightarrow A)$$

True ***no matter what*** the truth values of *A* and *B* are

Predicate calculus validity

$$\forall x \quad [[P(x) \rightarrow Q(x)] \vee [Q(x) \rightarrow P(x)]]$$

A fully quantified expression φ of predicate logic is a theorem *if and only if* φ is true for every possible meaning of each of the predicates of φ.

Theorem or not?

$$[\forall x \in \mathbb{Z} \quad P(x)] \vee [\forall x \in (Z) \neg P(x)]$$

(Not: consider P(x) = "x is a prime")

There's no algorithm that's guaranteed to figure out whether a given fully quantified expression φ is a theorem!
(Goedel's incompleteness theorem, Turing's undecidability of the halting problem)

Arguments with Quantified Statements

Universal instantiation:

$$\forall x, P(x)$$
$$\therefore P(a)$$

$$[\forall x \quad P(x)] \rightarrow P(a)$$

Universal modus ponens:

$$\forall x, P(x) \rightarrow Q(x)$$
$$P(a)$$
$$\therefore Q(a)$$

$$[[\forall x \quad P(x) \rightarrow q(x)] \wedge P(a)] \rightarrow Q(a)$$

Universal modus tollens:

$$\forall x, P(x) \rightarrow Q(x)$$
$$\neg Q(a)$$
$$\therefore \neg P(a)$$

Universal Generalization

valid rule

$$\frac{A \rightarrow R(c)}{A \rightarrow \forall x.R(x)}$$

providing *c* is independent of *A*

e.g. given any number c, 2c is an even number

=> for all x, 2x is an even number.