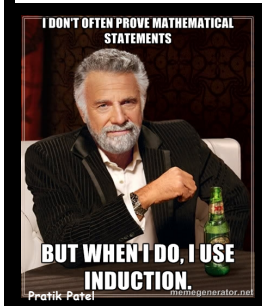
**``I think some intuition leaks out in every step of an induction proof''**

-- Jim Propp, talk at
American Mathematical Society special session,
1/22/00




# CS151 Fall 2014
# Lecture 11 – 9/30
# Induction

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http://www.cs.uic.edu/CS151

Adapted from Lap Chi Lau - The Chinese University of Hong Kong
and David Liben-Nowell - Carleton College

# Announcements

- **Labs will be consolidated (probably 3pm canceled). Look for announcements**
- **Google Tech Applications open for Full-Time and Internships**

The Engineering Practicum application targets freshmen and sophomores.
For our **Software Engineering full-time** opportunities, check out
g.co/SWEgrad

For our **Software Engineering internship** opportunities, check out
g.co/SWEintern

For our **Associate Product Manager full-time** opportunities, check out
g.co/APMgrad

For our **Associate Product Manager internship** opportunities, check out
g.co/APMintern

## Binary Search

I have a number between 0 and 63. You ask a question, I'll tell you yes or no.

How long will it take you to find my secret number?

```
BinarySearch (0…n-1)
middle = floor((n-1)/2)
if (middle == "secret number")
        return (middle)
else if (middle > "secret number")
                BinarySearch(0..middle)
    else
                BinarySearch(middle+1..n-1)
```

## Postage by Strong Induction

Available stamps:

5¢ 3¢

What amount can you form?

*Theorem*: Can form any amount ≥ 8¢

Prove by strong induction on $n$.

$P(n)$ ::= can form $(n+8)$¢.

## Postage by Strong Induction

**Base case** ($n$ = 0):

($0$ +8)¢:

**Inductive Step:** assume ($m$ +8)¢ for 0 <= $m$ <= $n$-1,
then prove ($n$ + 8)¢

**cases:**

$n$ = 1, 9¢:

$n$ = 2, 10¢:

## Postage by Strong Induction

**case** $n \geq 3$: let $m = n - 3$.

now $n - 1 \geq m \geq 0$, so by induction hypothesis have:

$(n-3)+8$ = $n+8$

We're done!

In fact, use at most two 5-cent stamps!

## Postage by Strong Induction

Given an unlimited supply of 5 cent and 7 cent stamps,

what postages are possible?

Theorem: For all n >= 24,

it is possible to produce n cents of postage from 5¢ and 7¢ stamps.

## Factorial

How many multiplication operations will it take to compute ***n!*** ?

n! = 1*2*3*…*(n-2)*(n-1)*n

```
Factorial (n)
if (n == 0 or n== 1)
        return(1)
else
        return (n*Factorial(n-1))
```

```
Factorial (n)
i := 1
factorial := 1
while i < n
        i := i+1
        factorial := factorial * i
return(factorial)
```

Factorial(n) returns ***n!***