

CS151 Fall 2014
Lecture 12 – 10/2

Recursive/Inductive
Definitions

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Adapted from Lap Chi Lau - The Chinese University of Hong Kong
and David Liben-Nowell - Carleton College

Factorial

How many multiplication operations will it take to compute $n!$?
 $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2) \cdot (n-1) \cdot n$

<pre> Factorial (n) if (n == 0 or n == 1) return(1) else return (n*Factorial(n-1)) </pre>	<pre> Factorial (n) i := 1 factorial := 1 while i < n i := i+1 factorial := factorial * i return(factorial) </pre>
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Factorial(n) returns $n!$

$P(n) :=$ " _____ "

Base Case: $P(?) :=$ " _____ "

$P(n-1) :=$

$P(n) :=$

Factorial

How many multiplication operations will it take to compute $n!$?
 $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2) \cdot (n-1) \cdot n$

<pre> Factorial (n) if (n == 0 or n == 1) return(1) else return (n*Factorial(n-1)) </pre>	<pre> Factorial (n) i := 1 factorial := 1 while i < n i := i+1 factorial := factorial * i return(factorial) </pre>
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Factorial(n) uses _____ multiplication operations

Recursive (Inductive) Definitions

We completely understand the function $\text{Factorial}(n) = n!$, right?
 As a reminder, here's the definition:
 $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n, n \geq 1$

But equivalently, we could define it like this:

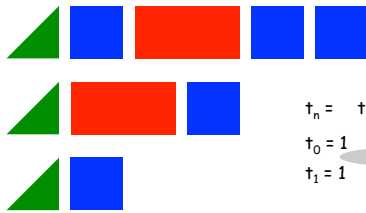
<div style="border: 1px solid gray; border-radius: 50%; padding: 5px; display: inline-block;">Recursive Case</div>	{	$n \cdot \text{Fac}(n-1)$	if $n > 1$
<div style="border: 1px solid gray; border-radius: 50%; padding: 5px; display: inline-block;">Base Case</div>		1,	if $n = 1$

Inductive
(Recursive)
Definition

Recurrences

A train is defined to be an engine, followed by cars of two different kinds...long and short. Long cars are 2 units long, and short ones are 1.

Examples:



How many trains of length n are there?

$$t_n = t_{n-1} + t_{n-2} \quad \text{Recursive Case}$$

$$t_0 = 1 \quad \text{Base Case}$$

$$t_1 = 1$$

We still don't really know.