http://xkcd.com/982/

CS151 Fall 2014
Lecture 15 – 10/16
Set Theory

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Sets

A ***set*** is a collection of mathematical objects,
with the collection treated as a single mathematical object.

Examples:
- real numbers, $\mathbb{R}$
- complex numbers, $\mathbb{C}$
- integers, $\mathbb{Z}$
- empty set, $\emptyset$

Defining Sets

Sets can be defined directly:

e.g. {1,2,4,8,16,32,…},

{CS101,CS151,…}

Order, number of occurence are not important.

e.g. {A,B,C} = {C,B,A} = {A,A,B,C,B}

A set can be an element of another set.

{1, {2}, {3,{4}} }

Defining Sets by Predicates

The set of elements, *x*, in *A* such that *P*(*x*) is true.

$$\{x \in A \mid P(x)\}$$

$$\{-1,0,1,2,3,4\} = \{x \in Z \mid -2 < x < 5\}$$

$$\{x \in R \mid -2 < x < 5\}$$

The set of prime numbers:

$$\{p \in Z \mid (p > 1) \wedge (\forall a \in Z > 1\ \forall b \in Z > 1\ a \cdot b \neq p)\}$$

Membership

{7, "Albert", $\pi/2$, T}

$x \in A$ — x is an element of A; x is in A

Examples:

$\pi/2 \in$ {7, "Albert", $\pi/2$, T}

$\pi/3 \notin$ {7, "Albert", $\pi/2$, T}

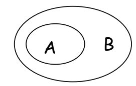
$14/2 \in$ {7, "Albert", $\pi/2$, T}

Containment

$A \subseteq B$ — A is a subset of B; A is contained in B

Every element of A is also an element of B.

$A \subseteq B \iff \forall x,$ if $x \in A$ then $x \in B$



$A \not\subseteq B \iff \exists x, x \in A$ and $x \notin B$

$A \subset B$ — A is a proper subset of B

$A = B \iff A \subseteq B$ and $B \subseteq A$

Examples

$\mathbb{Z} \subset \mathbb{R}$

$\{3\} \subset \{5,7,3\}$

$\emptyset \subseteq$ every set,

$A \subseteq A$

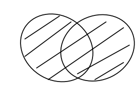
prime $\not\subseteq$ odd

{a, {b}, c} $\not\subseteq$ {a, b, c}

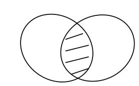
{{b}} $\neq$ {b}

Basic Operations on Sets

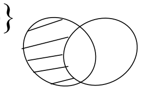
union: $A \cup B ::= \{x \mid (x \in A) \vee (x \in B)\}$



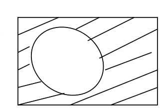
intersection: $A \cap B ::= \{x \mid x \in A \wedge x \in B\}$



difference: $A - B ::= \{x \mid (x \in A) \wedge (x \notin B)\}$



complement: $\overline{A} ::= \{x \in D \mid x \notin A\} = D - A$



Examples

A = {1, 3, 6, 8, 10}   B = {2, 4, 6, 7, 10}

$A \cap B$ = {6, 10}, $A \cup B$ = {1, 2, 3, 4, 6, 7, 8, 10}   A-B = {1, 3, 8}

prime $\cap$ even = {2},   even $\cap$ odd = $\emptyset$

A = { x | x = 2k for some integer k},   B = { x | x = 3k for some integer k}

$A \cap B$ = { x | x = 6k for some integer k}

$A \cup B$ = { x | x is either a multiple of 2 or a multiple of 3 (or both) }

A – B = { x | x is even but not a multiple of 3 }

Let D be the set of integers, then $\overline{A}$ = odd

Partitions of Sets

Two sets are disjoint if their intersection is empty.

A collection of nonempty sets $\{A_1, A_2, \ldots, A_n\}$ is a partition of a set A if and only if

$$A = A_1 \cup A_2 \cup \cdots \cup A_n$$

$A_1, A_2, \ldots, A_n$ are mutually disjoint.

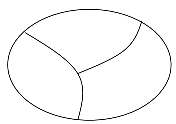
Example. Let A be the set of integers.

A_1 be the set of integers $\equiv$ 1 (mod 3)

A_2 be the set of integers $\equiv$ 2 (mod 3)

A_3 be the set of integers $\equiv$ 0 (mod 3)

$\{A_1, A_2, A_3\}$ is a partition of A



Power Sets

power set: $$\text{pow}(A) ::= \{S \mid S \subseteq A\}$$

pow({a,b}) = {$\emptyset$, {a}, {b}, {a,b}}

pow({a,b,c}) = {$\emptyset$, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}

pow({a,b,c,d}) = {$\emptyset$, {a}, {b}, {c}, {d},
{a,b}, {a,c}, {b,c}, {a,d}, {b,d}, {c,d},
{a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, {a,b,c,d}}

If A has n elements, then the pow(A) has 2^n elements.