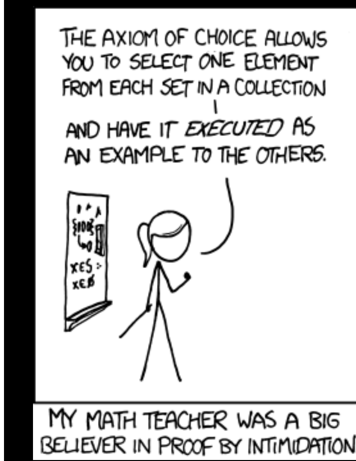
Announcements

- Title: Randomness
  Speaker: Avi Wigderson
  Time and Place: **TODAY** 4:00PM Lecture Center C, Room C4
- Recursion tutoring system
  instead of labs Monday October 20, 2-5 pm
  in **2254 SEL** (right across from 2249/2249F SEL)
  1-2pm lab will meet in regular place
- Grace Hopper Celebration Panel
  Tuesday October 21 @ 4:30pm in 1047 ERF
- 2nd Women in CS meeting
  Wednesday October 22nd @ 5 PM in SEO 1000



http://xkcd.com/982/

CS151 Fall 2014
Lecture 15 – 10/16
Set Theory

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/CS151

Adapted from Lap Chi Lau - The Chinese University of Hong Kong

Power Sets

power set: $\text{pow}(A) ::= \{S \mid S \subseteq A\}$

pow({a,b}) = {∅, {a}, {b}, {a,b}}

pow({a,b,c}) = {∅, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}

pow({a,b,c,d}) = {∅, {a}, {b}, {c}, {d},
{a,b}, {a,c}, {b,c}, {a,d}, {b,d}, {c,d},
{a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, {a,b,c,d}}

If A has n elements, then the pow(A) has 2^n elements.

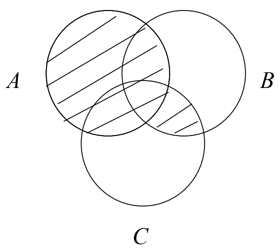
Claim: If A has n elements, then the pow(A) has 2^n elements.

Proof:

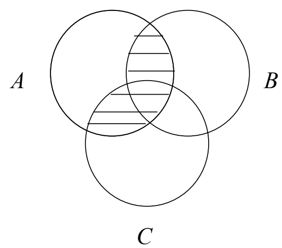
Set Identities

Distributive Law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (1)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (2)



(1)

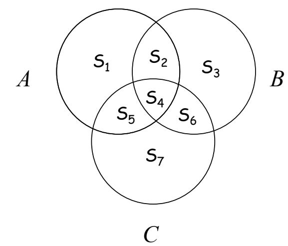


(2)

Set Identities

Distributive Law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

If you insist on proving this law more formally…



$A = S_1 \cup S_2 \cup S_4 \cup S_5$

$B \cap C = S_4 \cup S_6$

$A \cup (B \cap C) = S_1 \cup S_2 \cup S_4 \cup S_5 \cup S_6$

$(A \cup B) = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$

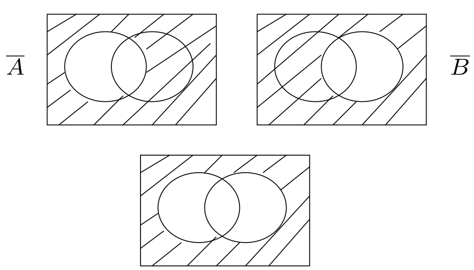
$(A \cup C) = S_1 \cup S_2 \cup S_4 \cup S_5 \cup S_6 \cup S_7$

$(A \cup B) \cap (A \cup C) = S_1 \cup S_2 \cup S_4 \cup S_5 \cup S_6$

There are even more formal proofs in the textbook…

Set Identities

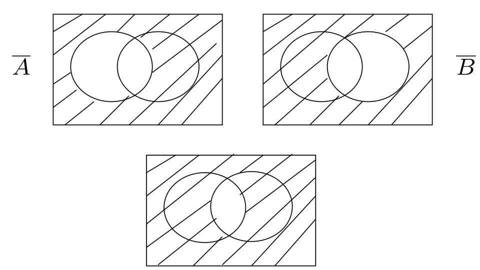
De Morgan's Law: $\overline{A \cup B} = \overline{A} \cap \overline{B}$



$\overline{A \cup B} = \overline{A} \cap \overline{B}$

Set Identities

De Morgan's Law: $\overline{A \cap B} = \overline{A} \cup \overline{B}$



$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Exercises

$A - (A \cap B) = A - B?$

$(A - B) \cup (B - C) = A - C?$

$(A \cup B) - C = (A - C) \cup (B - C)?$

Russell's Paradox

Let $W ::= \{S \in \text{Sets} \mid S \notin S\}$

so $S \in W \leftrightarrow S \notin S$

Is W in W?