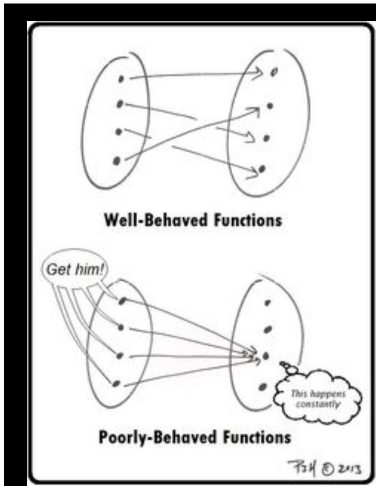


CS151 Fall 2014
Lecture 16 – 10/21
Functions

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/CS151

Adapted from Lap Chi Lau – The Chinese University of Hong Kong

Announcements

- **Grace Hopper Celebration Panel**
  Tuesday October 21 @ 4:30pm in 1047 ERF
- **2nd Women in CS meeting**
  Wednesday October 22nd @ 5 PM in SEO 1000
- **ACM research talk (Tanya Berger-Wolf)**
  Thursday October 23 @ 5pm in SEO 1000
- **Advising sign-up is this week** (October 20-24) and **advising is next week** (October 27-31).

  All undergraduate students have received e-mails from me and the college office about this. But, we have issues with students either forgetting about it or not understanding the importance (especially new students). So, please announce in your classes about advising and that they should check their e-mails or stop by our office (905 SEO).

  Advising list is posted on-line on CS site at http://www.cs.uic.edu/Main/UndergraduatePrograms.

Functions

$$f : A \rightarrow B$$

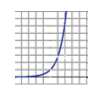
function, f, from set A to set B
associates an element $f(a) \in B$, with an element $a \in A$.

The *domain* of f is A.
The *codomain/range* of f is B.

For every input there is exactly one output.

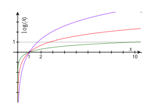
Functions

$f(x) = e^x$



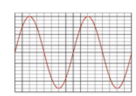
domain = R
range = R+-{0}

$f(x) = \log(x)$



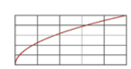
domain = R+-{0}
range = R

$f(x) = \sin(x)$



domain = R
range = [0,1]

$f(x) = \sqrt{x}$



domain = R+
range = R+

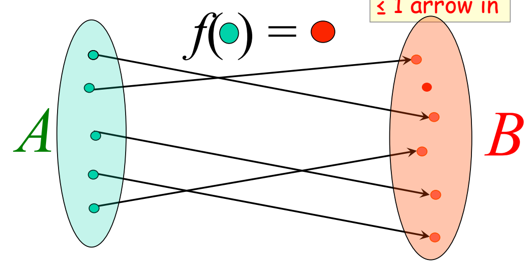
Functions

f(S) = |S|	domain = the set of all sets range = non-negative integers
f(string) = length(string)	domain = the set of all strings range = non-negative integers
f(student-name) = student-ID	not a function, since one input could have more than one output
f(x) = is-prime(x)	domain = positive integers range = {T,F}

Injections (One-to-One)

$f : A \rightarrow B$ is an *injection* iff no two inputs have the same output.

≤ 1 arrow in



$\forall a, a' \in A.$

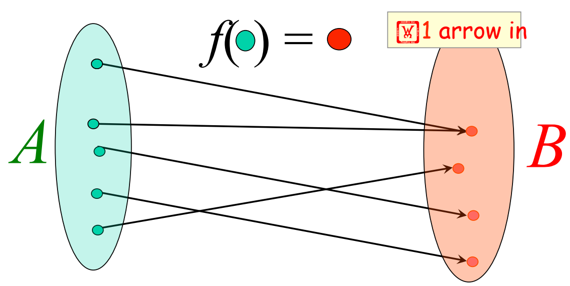
$(f(a) = f(a')) \rightarrow (a = a')$

$|A| \leq |B|$

Surjections (Onto)

$f : A \rightarrow B$ is a *surjection* iff every output is possible.

≥1 arrow in



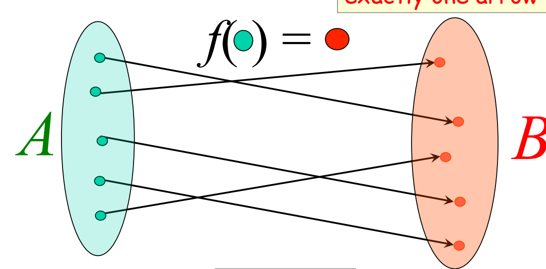
$\forall b \in B\, \exists a \in A.\, f(a) = b$

$|A| \geq |B|$

Bijections

$f : A \rightarrow B$ is a *bijection* iff it is surjection and injection.

exactly one arrow in



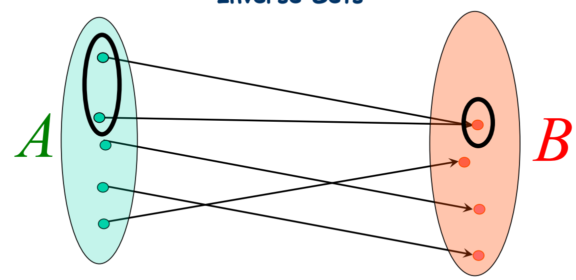
$|A| = |B|$

In-Class Exercises

Function	Domain	Codomain	Injective?	Surjective?	Bijective?
f(x)=sin(x)	Real	Real	No	No	No
$f(x)=2^x$	Real	Positive real	Yes	Yes	Yes
$f(x)=x^2$	Real	Non-negative real	No	Yes	No
Reverse string	Bit strings of length n	Bit strings of length n	Yes	Yes	Yes

Whether a function is injective, surjective, bijective depends on its domain, codomain and the range.

Inverse Sets

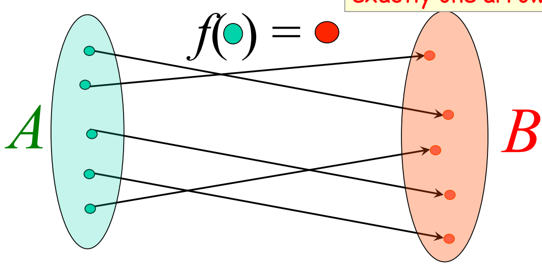


Given an element y in B, the inverse set of y := $f^{-1}(y) = \{x \text{ in } A \mid f(x) = y\}$.

Inverse Function

Informally, an inverse function f^{-1} is to "undo" the operation of function f.

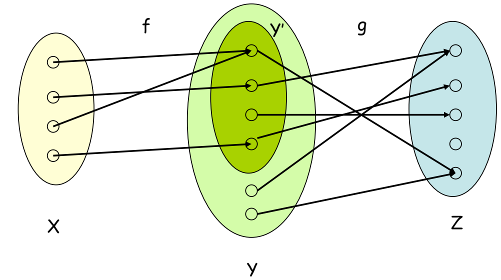
exactly one arrow in



There is an inverse function f^{-1} for f if and only if f is a bijection.

Composition of Functions

Two functions f:X->Y', g:Y->Z so that Y' is a subset of Y,
then the composition of f and g is the function $g_\circ f$: X->Z, where
$g_\circ f(x) = g(f(x))$.

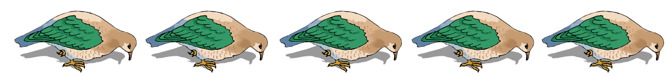


In-Class Exercises

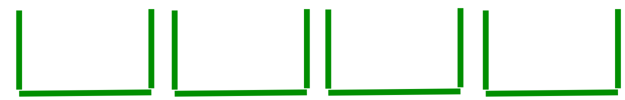
Function f	Function g	g∘f injective?	g∘f surjective?	g∘f bijective?
f:X->Y f surjective	g:Y->Z g injective	No	No	No
f:X->Y f surjective	g:Y->Z g surjective	No	Yes	No
f:X->Y f injective	g:Y->Z g surjective	No	No	No
f:X->Y f bijective	g:Y->Z g bijective	Yes	Yes	Yes
f:X->Y	f^{-1}:Y->X	Yes	Yes	Yes

Pigeonhole Principle

If more pigeons

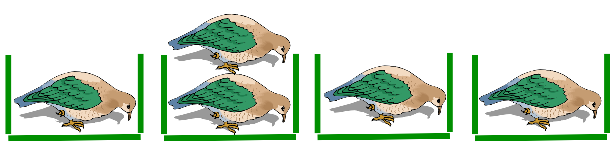


than pigeonholes,



Pigeonhole Principle

then some hole must have at least two pigeons!



Pigeonhole principle

A function from a larger set to a smaller set cannot be injective.

(There must be at least two elements in the domain that have the same image in the codomain.)

Example

Question: Let A = {1,2,3,4,5,6,7,8}

If five integers are selected from A, must a pair of integers have a sum of 9?

Consider the pairs {1,8}, {2,7}, {3,6}, {4,5}.
Among them, they contain all the numbers in A
The sum of each pair is equal to 9.

The pairs are the holes.

If we choose 5 numbers from the set A,

(those numbers are the pigeons)

then by the pigeonhole principle,

both elements of some pair will be chosen,

and their sum is equal to 9.