## Slide 1

CS151 Fall 2014
Lecture 20 – 11/4

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/bin/view/CS151/WebHome

Combinatorics:
Still Counting

1...2...
BAAA

...1,306... 1,307...
BAAA

...32,767...-32,768...
BAAA BAAAA

...-32,767...-32,766...
BAAA

Rundall Munroe: xkcd.com/571/

It's actually a stack overflow joke,
not counting, but close enough

## Slide 2

### Announcements

**VOTE TODAY!**

**Howard Tullman, CEO, 1871**
**"You Don't Want to Be At The Airport When Your Ship Comes In"**
**Thursday, November 6, 2014, 10:30 a.m. Room 1000 SEO**
A rapid fire review of the six major trends at the intersection of social media and technology which are radically changing the ground rules and the ways in which every business will reach and engage its customer from now on.

**CS Scavenger Hunt**
**Thursday Nov 6** SEO 1000
4:45pm early bird drawing and team assignments
5pm the hunt begins....
6pm FREE (good) Food and Prizes
Please RSVP to elathos@uic.edu

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## Slide 3

### Practice with Permutations

- What is P(5,2)?

- How many 4-permutations are there of a set of 7 objects?

- How many 5-permutations are there of set of 5 objects?

## Slide 4

### Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.
How many different Top 10 lists can you make?

P(17,10) =
17x16x15x14x13x12x11x10x9x8

### Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.

Now suppose you want to have 4 animal, 4 car, and 2 sports results in the Top 10, in that band order. How many Top 10 lists can you make?

$$P(6,4) \times P(8,4) \times P(3,2)$$

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### Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.

Now suppose you want to have 4 animal, 4 car, and 2 sports results in the Top 10, **the order doesn't matter**, but you want the results of the same type to appear together. How many Top 10 lists can you make?

$$P(6,4) \times P(8,4) \times P(3,2) \times 3!$$

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### Permutations

In how many ways can 5 distinct Martians and 3 distinct Jovians stand in line, if no two Jovians stand together?

____ M1 ____ M2 ____ M3 ____ M4 ____ M5 ____

$$5! \times P(6,3)$$

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### Disjoint Sets: Addition Rule

How many PINs of length **at most 4** are there with letters, numbers, repetitions allowed, not case sensitive?

| PINS of length 1 | PINS of length 2 | PINS of length 3 | PINS of length 4 |
|---|---|---|---|
| 36 | $36^2$ | $36^3$ | $36^4$ |

(with $+$ between each term)

Suppose a finite set A equals the union of k distinct mutually disjoint subsets $A_1$, $A_2$, ..., $A_k$. Then

$$N(A) = N(A_1) + N(A_2) + \dots + N(A_k) = \sum_{i=1}^{k} N(A_i)$$

## Disjoint Sets: Addition Rule

•How many three digit integers are divisible by 5?
End with 0 + end with 5

•How many times is the statement *Statement* executed?
for *i=1* to *n*
for *j=1* to *i*
*Statement*

## Difference Rule

How many PINs of length **exactly 4** are there with letters, numbers, repetitions allowed, not case sensitive? How many contain repeated symbols?
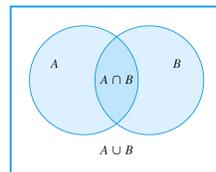
$$36\char`^4 – P(36,4) =$$
$$36\char`^4 – 36!/32! =$$
$$36\char`^4 –$$
$$36*35*34*33$$

If A is a finite set and B is a subset of A then
N(A-B) = N(A) - N(B)

## Inclusion/Exclusion

How many integers from 1 to 1,000 are multiples of 15?
How many multiples of either 3 or 5?



If A, B, and C are any finite sets, then

$$N(A \cup B) = N(A) + N(B) – N(A \cap B)$$

and

$$N(A \cup B \cup C) = N(A) + N(B) + N(C)$$
$$– N(A \cap B) – N(A \cap C) – N(B \cap C) + N(A \cap B \cap C)$$