

CS151 Fall 2014 Lecture 21 - 11/6

Combinatorics: Still Counting

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Combinations

A combination is an unordered selection of elements from some set.

The number of combinations of r distinct objects chosen from n distinct objects is denoted by $C(n,r)$ or nCr or $\binom{n}{r}$, and is read "n choose r."

$$C(n,r) = P(n,r)/r! = n!/((n-r)!r!)$$

Combinations

A basketball squad consists of 12 players, 5 of which make up a team. How many different teams of players can you make from the 12?

$$C(12,5)$$

What's the diff?

In a running race of 12 sprinters, each of the top 5 finishers receives a different medal. How many ways are there to award the 5 medals?

$$P(12,5) = C(12,5) \times 5!$$

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can 3 frosh and 5 soph be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **exactly** 1 frosh be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **at most** 1 frosh be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **at least** 1 frosh be selected?

Combinations

- How many 8 bit strings with exactly three 1's?
- How many distinct words are there from MISSISSIPPI?

Suppose a collection consists of n objects of which
 n_1 are of type 1 and are indistinguishable from each other
 n_2 are of type 2 and are indistinguishable from each other
 \dots
 n_k are of type k and are indistinguishable from each other
 and $\sum_{i=1}^k n_i = n$. Then the number of distinct permutations of the n objects is

$$\binom{n}{n_1} \binom{n-n_1}{n_2} \binom{n-n_1-n_2}{n_3} \dots \binom{n-n_1-n_2-\dots-n_{k-1}}{n_k} = \frac{n!}{n_1!n_2!n_3!\dots n_k!}$$

Combinations with Repetitions Allowed

How many multisets of size three are there from {1,2,3,4}?

- {1,1,1} {1,1,2} {1,1,3} {1,1,4}
- {1,2,2} {1,2,3} {1,2,4}
- {1,3,3} {1,3,4} {1,4,4}
- {2,2,2} {2,2,3} {2,2,4}
- {2,3,3} {2,3,4} {2,4,4}
- {3,3,3} {3,3,4} {3,4,4}
- {4,4,4}

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Category 1	Category 2	Category 3	Category 4	Result of Selection
x x x				{1, 1, 1}
x		x	x	{1, 3, 4}
	x x		x	{2, 2, 4}

Arranging 3 x and 3 | in the 6 positions. Choose where to put x and put the | in the remaining positions: $C(6,3) = 6!/(3!3!) = 20$

\$\$\$||| {1,1,1}
 |\$\$|\$\$ {2, 2, 4}