

**CS151 Fall 2014
Lecture 22 - 11/11**

**Combinatorics:
More Counting**

Prof. Tanya Berger-Wolf
<http://www.cs.ucsb.edu/bin/view/CS151/WebHome>

<http://spikedmath.com/368.html>

Announcements

DISTINGUISHED LECTURER SERIES
 The Evolution of Malware: From Invasion to Evasion
 Dr. Giovanni Vigna
 University of California, Santa Barbara
Thursday, November 13, 2014
 11:00 a.m., 1000 SEO Building

CS Inside Scoop Panel, Monday November 17th @ 5 PM
 The WiCS Inside Scoop Panel is open to all students and will be an open panel about course selections. This is the time to ask about which courses are good to take together, which courses are just awful, which professors are great for specific courses and more! The Panel will be made up of students who have taken a significant amount of 300-400 level courses.
 If you are interested please RSVP at <http://bit.ly/149yc2v>

Combinations with Repetitions Allowed

How many multisets of size three are there from {1,2,3,4}?

{1,1,1} {1,1,2} {1,1,3} {1,1,4}
 {1,2,2} {1,2,3} {1,2,4}
 {1,3,3} {1,3,4} {1,4,4}
 {2,2,2} {2,2,3} {2,2,4}
 {2,3,3} {2,3,4} {2,4,4}
 {3,3,3} {3,3,4} {3,4,4}
 {4,4,4}

20

| Category 1 | Category 2 | Category 3 | Category 4 | Result of Selection |
|------------|------------|------------|------------|---------------------|
| x x x | | | | {1, 1, 1} |
| x | | x | x | {1, 3, 4} |
| | x x | | x | {2, 2, 4} |

Arranging 3 x and 3 | in the 6 positions. Choose where to put x and put the | in the remaining positions: $C(6,3) = 6!/(3!3!) = 20$

Combinations with Repetitions Allowed

How many solutions to the equation $x_1 + x_2 + x_3 = 11$ for non-negative integers?

| Category 1 x_1 | Category 2 x_2 | Category 3 x_3 | Result of Selection |
|---------------------|---------------------|---------------------|---------------------|
| xxxxxxx | | xxxx | 7 + 0 + 4 |
| xx | xxxxxxx | xxx | 2 + 6 + 3 |
| | xxxxxxxxxxx | x | 0 + 10 + 1 |

Arranging 11 x and 2 | in the 13 positions. Choose where to put | and put the x in the remaining positions: $C(13,2) = 13!/(2!11!) = 78$

Combinations with Repetitions Allowed

How many solutions to the equation $x_1 + x_2 + x_3 = 11$ for positive integers?

| Category 1 x_1 | Category 2 x_2 | Category 3 x_3 | Result of Selection |
|---------------------|---------------------|---------------------|---------------------|
| xxxxx x | x | xxx x | 6 + 1 + 4 |
| x x | xxxxx x | xx x | 2 + 6 + 3 |
| x | xxxxxxxx x | x | 1 + 9 + 1 |

Put one x into each category to ensure it is non-zero now the remaining 8 x and 2 | can be arranged in $C(10,2) = 45$ ways

Summary

- n different objects in n spaces, order important $n!$
- n different objects in r spaces, order important $P(n,r) = n!/(n-r)!$
- n different objects in r spaces, order not important $C(n,r) = \binom{n}{r} = n! / [(n-r)!r!] = P(n,r)/r!$

Multiplication Rule: object consists of a **sequence** of independent choices (AND)

Addition Rule: object consists of a **collection** of independent choices (OR)

Binomial Coefficients

$$(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$$

$$= \binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$$

Binomial Theorem: Let x and y be variables, and let n be any nonnegative integer. Then

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j$$

Binomial Coefficients

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What is the coefficient of a^8b^9 in the expansion of $(3a + 2b)^{17}$?

What is n? 17

What is j? 9 $\binom{17}{9} (3a)^8 (2b)^9 = \binom{17}{9} 3^8 2^9 a^8 b^9$

What is x? 3a

What is y? 2b

Binomials: Application Example

· How many PINs (of letters and numbers) of length 4 contain 3 letters and 1 number?

· $36^4 - (26+10)^4 = 26^3 \cdot 10$

· For a query "zebra", a search engine returns 8,324,154 results about the animal and 46,537 results related to software and technology. If the results are unranked and repetitions are allowed, how many Top 10 lists of the results contain 7 animal and 3 technology results?