CS151 Fall 2014
Lecture 23 – 11/13

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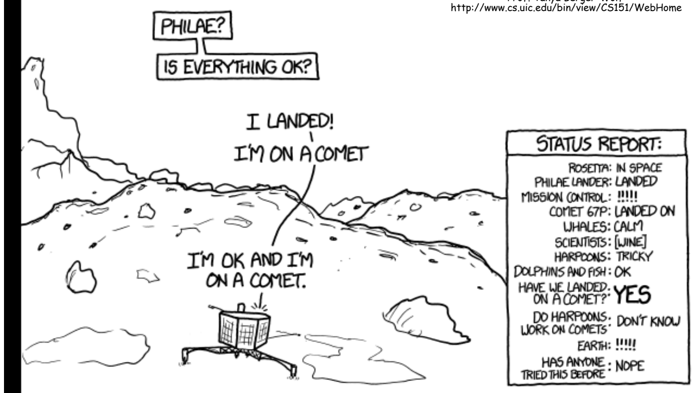
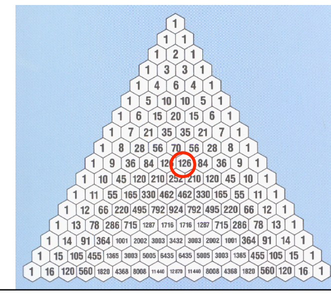
http://www.cs.uic.edu/bin/view/CS151/WebHome



Binomial Coefficients

$(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$

$$= \binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$$

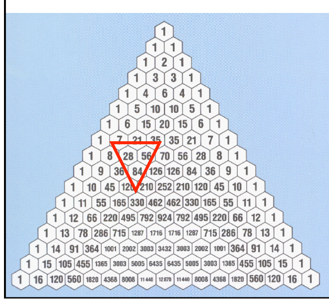


A. C(10,6)
B. C(9,4)
C. C(9,5)
D. C(8,4) + C(8,5)
E. No clue

Pascal's Identity

A relationship between the entries in Pascal's ▲.

$$\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}$$



Suppose T is a set, |T|=n. Let a be an element in T, and let S = T - {a}. Let's count the ${}_nC_j$ subsets of size j. Note that some of these contain a, and some don't.

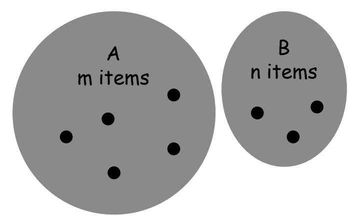
How many contain a? ${}_{n-1}C_{j-1}$

How many don't? ${}_{n-1}C_j$

Vandermonde's Identity

Let m, n, and r be nonnegative integers with r not exceeding either m or n. Then

$$\binom{m+n}{r} = \sum_{j=0}^{r} \binom{m}{r-j}\binom{n}{j}$$



To choose r items, take some from A and some from B. All possible ways of doing this gives the result.

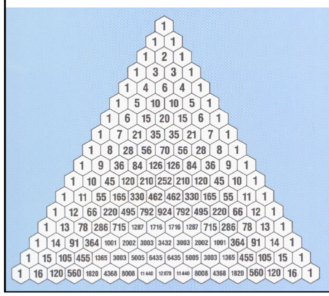
Binomial Coefficients

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

Sum each row of Pascal's Triangle:

Powers of 2

Two proofs that $\sum_{j=0}^{n} \binom{n}{j} = 2^n$



Suppose you have a set of size n. How many subsets does it have? 2^n

How many subsets of size 0 does it have? C(n,0)

How many subsets of size 1 does it have? C(n,1)

How many subsets of size 2 does it have? C(n,2)

Count all subsets in this way, and we have the result!

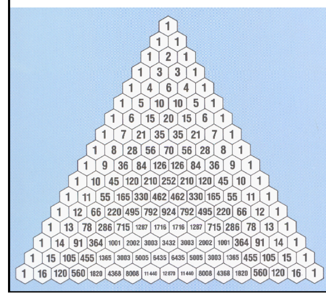
Binomial Coefficients

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

Sum each row of Pascal's Triangle:

Powers of 2

Two proofs that $\sum_{j=0}^{n} \binom{n}{j} = 2^n$



Let x=1 and y=1 in Binomial Theorem.

Done

$$\sum_{j=0}^{n} \binom{n}{j} 1^{n-j} 1^j = (1+1)^n$$

$$\sum_{j=0}^{n} \binom{n}{j} = 2^n$$