

**CS151 Fall 2014  
Lecture 23 - 11/13**  
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PHILAE?  
IS EVERYTHING OK?

I LANDED!  
I'M ON A COMET

I'M OK AND I'M ON A COMET.

**STATUS REPORT:**

ROSETTA: IN SPACE  
PHILAE LANDER: LANDED  
MISSION CONTROL: !!!!!  
COMET 67P: LANDED ON  
WHALES: CALM  
SCIENTISTS: [UNE]  
HARPOONS: TRICKY  
DOLPHINS AND FISH: OK  
HAVE WE LANDED ON A COMET? **YES**  
DO HARPOONS: DON'T KNOW  
WORK ON COMETS:  
EARTH: !!!!!  
HAS ANYONE: NOPE  
TRIED THIS BEFORE:

### Binomial Coefficients

$(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$

$$= \binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$$

A.  $C(10,6)$   
B.  $C(9,4)$   
C.  $C(9,5)$   
D.  $C(8,4) + C(8,5)$   
E. No clue

### Pascal's Identity

A relationship between the entries in Pascal's  $\triangle$ .

$$\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}$$

Suppose  $T$  is a set,  $|T|=n$ . Let  $a$  be an element in  $T$ , and let  $S = T - \{a\}$ . Let's count the  $\binom{n}{j}$  subsets of size  $j$ . Note that some of these contain  $a$ , and some don't.

How many contain  $a$ ?  $\binom{n-1}{j-1}$

How many don't?  $\binom{n-1}{j}$

### Vandermonde's Identity

Let  $m, n$ , and  $r$  be nonnegative integers with  $r$  not exceeding either  $m$  or  $n$ . Then

$$\binom{m+n}{r} = \sum_{j=0}^r \binom{m}{r-j} \binom{n}{j}$$

To choose  $r$  items, take some from  $A$  and some from  $B$ . All possible ways of doing this gives the result.

**Binomial Coefficients**

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j$$

Sum each row of Pascal's Triangle: Powers of 2

Two proofs that  $\sum_{j=0}^n \binom{n}{j} = 2^n$

Suppose you have a set of size  $n$ . How many subsets does it have?  $2^n$

How many subsets of size 0 does it have?  $C(n,0)$

How many subsets of size 1 does it have?  $C(n,1)$

How many subsets of size 2 does it have?  $C(n,2)$

Count all subsets in this way, and we have the result!

**Binomial Coefficients**

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j$$

Sum each row of Pascal's Triangle: Powers of 2

Two proofs that  $\sum_{j=0}^n \binom{n}{j} = 2^n$

Let  $x=1$  and  $y=1$  in Binomial Theorem. Done

$$\sum_{j=0}^n \binom{n}{j} 1^{n-j} 1^j = (1 + 1)^n$$

$$\sum_{j=0}^n \binom{n}{j} = 2^n$$