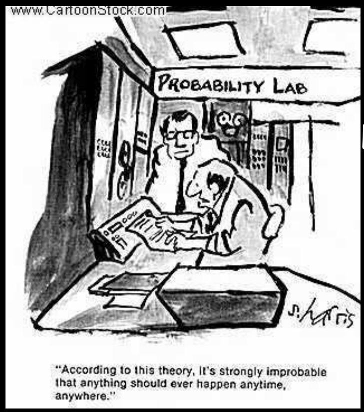

CS151 Fall 2014
Lecture 24– 11/18

Probability

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www.cs.uic.edu/~151

Announcements

- Final exam: Wed Dec 10, 3:30-5:30

  IF YOU HAVE CONFLICT – LET ME KNOW NOW!

Probability

Probability measure on a set S is a real-valued function, Pr, with domain 2^S so that:

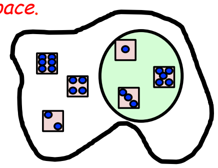
1. For any subset A in 2^S, $0 \le Pr(A) \le 1$.
2. $Pr(\emptyset) = 0$, $Pr(S) = 1$.
3. If subsets A and B are disjoint, then $Pr(A \cup B) = Pr(A) + Pr(B)$.

Pr(A) is "the probability of event A."

A sample space, together with a probability measure, is called a *probability space.*

$S = \{1,2,3,4,5,6\}$

For $A \subseteq S$, $Pr(A) = |A|/|S|$ *if all outcomes are equally likely*



Ex. "Prob of an odd #"

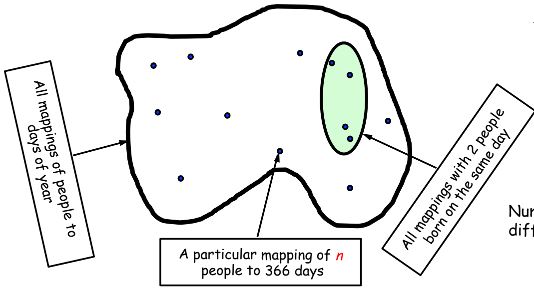
$A = \{1,3,5\}$, $Pr(A) = 3/6$

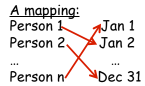
Probability: Examples

- What is the probability that a randomly chosen three digit integer is divisible by 5?
- If all four symbol PINs (letters and numbers) are equally likely, what is the probability that a randomly chosen PIN contains no repeated symbols?
- Suppose you have two dice: red and white. What is the probability that the number on the red die is greater than the number on the white?
- Among 300 people there are 142 ***pairs*** of friends. What is the probability that a randomly chosen pair of people are friends?

The Birthday Problem

With n people in a room, what is the probability that two have the same birthday?





366^n mappings

Number of ways to pick n different birthdays: $\frac{366!}{(366-n)!}$

Pr(No two birthdays on the same day) =(No two same birthdays)/(All mappings) = $\frac{\frac{366!}{(366-n)!}}{366^n}$

Pr(There are two people with same BDay) = 1 - Pr(No two BDays on same day)

Turns out, if there are 23 people then Pr(There are two people with same BDay) > 0.5

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Birthday paradox

Let's see if it's true for the class.

Start saying your birthday (no year). Raise your hand if you have the same birthday.

We have 6 pairs! What is the probability of that? (Homework 7 question)

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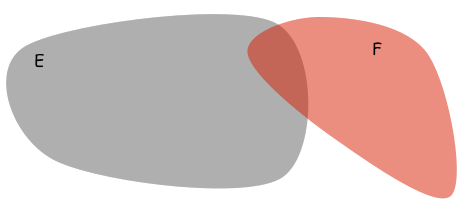
Conditional Probability

Let E and F be events with Pr(F) > 0. The conditional probability of E given F, denoted by Pr(E|F) is defined to be:

Pr(E|F) = Pr(E∩F)/Pr(F).



Conditional Probability

Pr(E|F) = Pr(E∩F)/Pr(F).

A bit string of length 4 is generated at random so that each of the 16 bitstrings is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0?

Pr(F) = 1/2

Pr(E∩F)? 0000 0001 0010 0011 0100

Pr(E∩F) = 5/16 Pr(E|F) = 5/8

Independence

The events E and F are *independent* if and only if Pr(E∩F) = Pr(E) x Pr(F).

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 2? No

$Pr(E) = \frac{1}{2}$, $Pr(F) = \frac{3}{4}$, $Pr(E \cap F) = \frac{1}{2}$

Independence

The events E and F are *independent* if and only if Pr(E∩F) = Pr(E) x Pr(F).

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 3? Yes

Pr(E) = 6/8 = 3/4, Pr(F) = 1/2, Pr(E∩F) = 3/8

Independence

The events E and F are *independent* if and only if Pr(E∩F) = Pr(E) x Pr(F).

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 4? No

Independence

The events E and F are *independent* if and only if Pr(E∩F) = Pr(E) x Pr(F).

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 5? No

Independence

The events E and F are *independent* if and only if Pr(E∩F) = Pr(E) x Pr(F).

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 2? No

n = 3? Yes

n = 4? No

n = 5? No