Relations

Recall the definition of the Cartesian (Cross) Product:

The Cartesian Product of sets A and B, A x B, is the set

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$.

aRb or **$(a,b) \in R$** means **"*a* is related to *b*"**

A relation is just any subset of the Cartesian Product:

$R \subseteq A \times B$

- Ex1: A = {0,1,2}, B = {2,3} => AxB = {(0,2), (0,3), (1,2), (1,3), (2,2), (2,3)}
  R = {(a,b) | a < b}. So R = {(0,2), (0,3), (1,2), (1,3), (2,3)} = AxB-{(2,2)}
- Ex2: A = students at UIC; B = courses at UIC.
  R = {(a,b) | student a is enrolled in class b}
- Ex3: A = {3 letter strings}, B = {all English words}
  R = {(a,b) | a is a prefix of b}

Relations and Functions

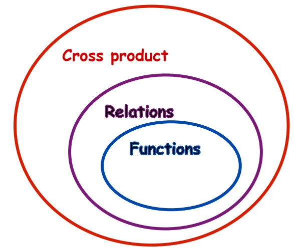
Recall the definition of a function:

$f = \{(a,b) : b = f(a) , a \in A \text{ and } b \in B\}$

Is every function a relation?

Yes, a function is a special kind of relation.

Draw Venn diagram of **cross products, relations, functions**



Properties of Relations

Reflexivity:

A relation R on AxA is reflexive if for all $a \in A$, $(a,a) \in R$.

Symmetry:

A relation R on AxA is symmetric if $(a,b) \in R$ implies $(b,a) \in R$.

Properties of Relations

Transitivity:

A relation on AxA is transitive if $(a,b) \in R$ and $(b,c) \in R$ imply $(a,c) \in R$.

Anti-symmetry:

A relation on AxA is anti-symmetric if $(a,b) \in R$ implies $(b,a) \notin R$.