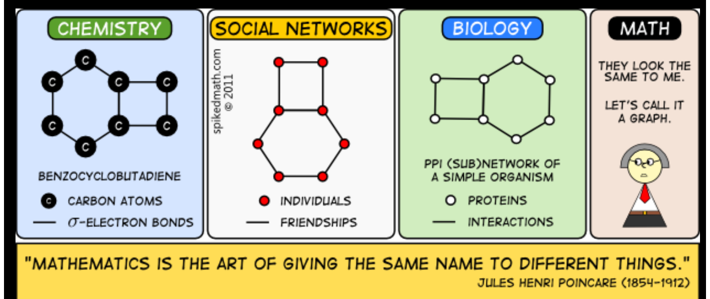
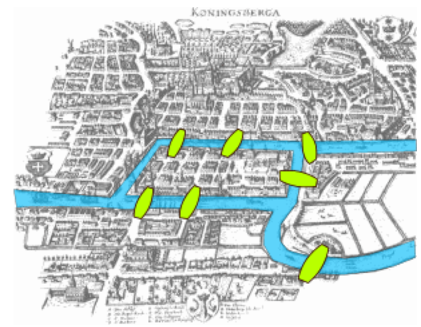
CS151 Fall 20124Lecture (almost there) – 12/6

Graphs



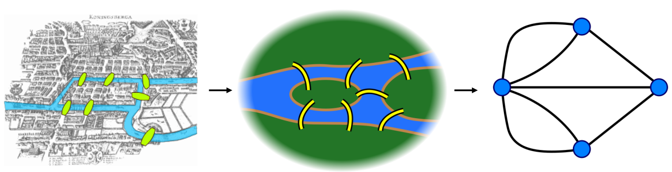
Seven Bridges of Königsberg



Leonard Euler
1707-1783

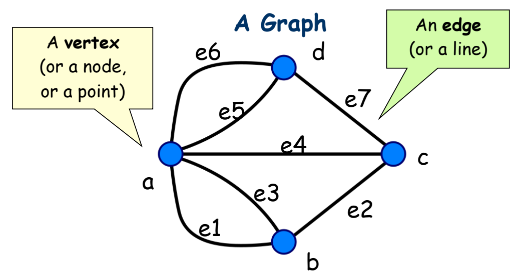
Is it possible to walk with a route that crosses each bridge

Seven Bridges of Königsberg



Forget unimportant details. Forget even more.

A Graph

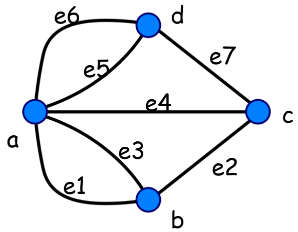


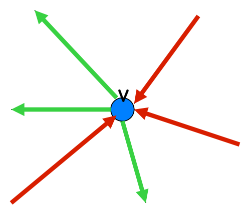
So, what is the **"Seven Bridges of Königsberg"** problem

To find a walk that visits each edge exactly once.

Euler's Solution

Question: Is it possible to find a walk that visits each edge



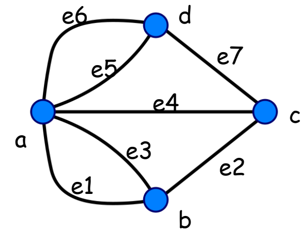


Suppose there is such a walk, there is a starting point and an

For every "intermediate" point v, there must be the same nu
incoming and outgoing edges, and so v must have an **even num**

Euler's Solution

Question: Is it possible to find a walk that visits each edge



So, at most **two** vertices can have odd number of edges.

In this graph, every vertex has only an odd number of edges, and so there is no walk which visits each edge exactly one.

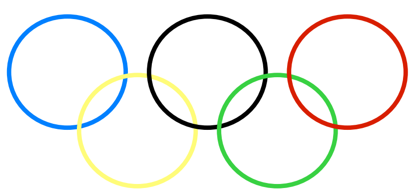
Suppose there is such a walk, th ... d an

For every "intermediate" point v, there must be the same nu
incoming and outgoing edges, and so v must have an **even num**

Euler's Solution

So Euler showed that the **"Seven Bridges of Königsberg"** i

When is it possible to have a walk that visits every edge ex

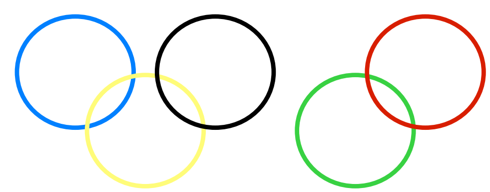


Is it always possible to find such a walk if there is **at most two** vertices with odd number of edges?

Euler's Solution

So Euler showed that the **"Seven Bridges of Königsberg"** i

When is it possible to have a walk that visits every edge ex



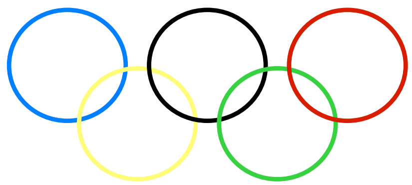
Is it always possible to find such a walk if th... is **at most two** vertices with odd number of edges?

NO!

Euler's Solution

So Euler showed that the **"Seven Bridges of Königsberg"** i

When is it possible to have a walk that visits every edge ex



Is it always possible to find such a walk if the graph i... **YES!**
and there are **at most two** vertices with odd number of edg

Euler's Solution

So Euler showed that the **"Seven Bridges of Königsberg"** i

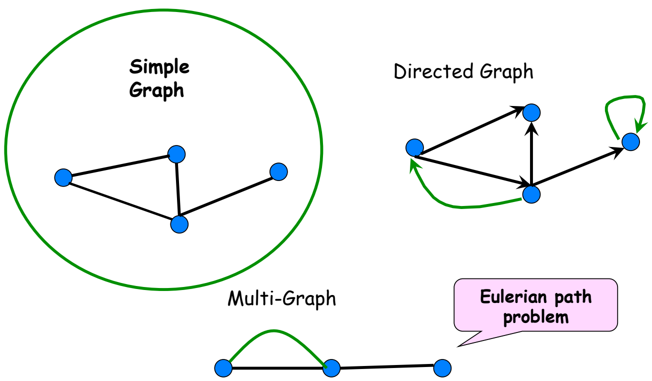
When is it possible to have a walk that visits every edge ex

Eulerian path

Euler's theorem: A graph has an Eulerian path if and only i
"connected" and has at most two vertices with an odd numb

This theorem was proved in 1736,
and was regarded as the starting point of graph theo

Types of Graphs



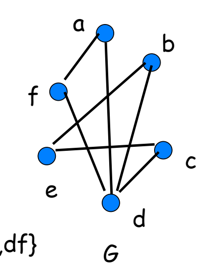
Simple Graphs

A graph G=(V,E) consists of:

A set of vertices, *V*

A set of *undirected* edges, *E*

- V(G) = {a,b,c,d,e,f}
- E(G) = {ad,af,bd,be,cd,ce,df}



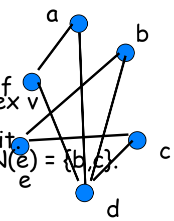
Two vertices a,b are ***adjacent*** (***neighbours***) if the edge ab

Vertex Degrees

An edge uv is ***incident*** on the vertex u and the vertex v.

The ***neighbour set*** N(v) of a vertex v is the set of vertices adjacent to it.

e.g. N(a) = {d,f}, N(d) = {a,b,c,f}, N(e) = {b,c}.



degree of a vertex = # of ***incident*** edges

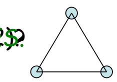
e.g. deg(d) = 4, deg(a)=deg(b)=deg(c)=deg(e)=deg(f)=2.

the degree of a vertex v = the number of neighbours o

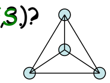
For multigraphs, NO. For simple graphs, YES.

Degree Sequence

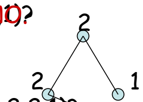
Is there a graph with degree sequence (2,2,2)? YES.



Is there a graph with degree sequence (3,3,3,3)? YES.



Is there a graph with degree sequence (2,2,1)? NO.



Is there a graph with degree sequence (2,2,2,2,1)?

Where to go?

NO. What's wrong with these sequences?

Handshaking Lemma

For any graph, sum of degrees = twice # edges

Lemma. $2|E| = \sum_{v \in V} \deg(v)$

Corollary.
1. Sum of degree is an even number.
2. Number of odd degree vertices is even.

Examples. 2+2+1 = odd, so impossible.
2+2+2+2+1 = odd, so impossible.

Handshaking Lemma

Lemma. $2|E| = \sum_{v \in V} \deg(v)$

Proof. Each edge contributes 2 to the sum on the Dig QED

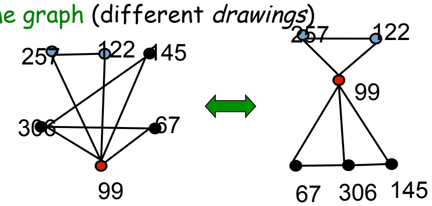
Question. Given a degree sequence, if the sum of degree is it true that there is a graph with such a degree seq

For simple graphs, NO, consider the degree sequence (3

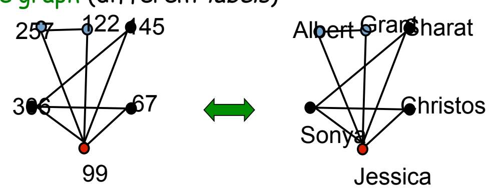
For multigraphs (with self loops), YES! (easy by induc

Same Graphs?

Same graph (different *drawings*)



Same graph (different *labels*)



Graph Isomorphism

All that matters is the *connections.*

Graphs with the same connections are

Informally, two graphs are isomorphic if they are the same c

G_1 *isomorphic* to G_2 means there is an *edge-preserving vertex matching.*

relation preserving

renaming function

Graph isomorphism has applications like checking fingerprint, t