CS202 Fall 2012
Lecture 3 – 9/4

Combinatorics: Still Counting

Prof. Tanya Berger-Wolf
http://www.cs.uic.edu/bin/view/CS202/WebHome

Announcements

Enforced prerequisites:

Course Information: 3 hours. Previously listed as EECS 360. Prerequisite(s): Grade of C or better in CS 201.

Registration Restriction: Restricted to Engineering or School of Continuing Studies.

Grace Hopper Conference for Women in Computing
Scholarships available
If you would like to participate email me: tanyabw@uic.edu

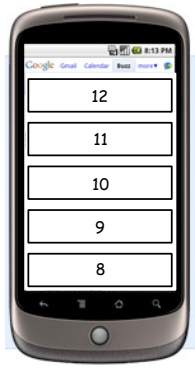
Counting!

- How many 4 symbol PINs are there with letters, numbers, repetitions allowed, not case sensitive?
- PINs without repetitions?
- Bit strings of length 5?
- Bit strings of length 5 that start and end with the same bit?
- Iterations of a nested loop? For i=1..n For j= 1..m ….
- 10 pages ranked 1st. How many orderings are there of those ten pages?
- Ways the letters of the word COMPUTER can be arranged in a row?

For any non-negative integer *n*, the number of permuitations of a set with *n* elements is *n!*

Permutations of Selected Elements

A search engine returns 12 top ranked results but on the mobile interface only 5 can appear on the first page. How many ways are there to display the results of the first page



a) 60
b) 12^5
c) 12!/7!
d) 5^{12}
e) No clue

A permutation is an ordered arrangement of objects.

The number of permutations of r distinct objects chosen from n distinct objects is denoted P(n,r).

P(n,r) = n! / (n-r)!

Practice with Permutations

- What is P(5,2)?
- How many 4-permutations are there of a set of 7 objects?
- How many 5-permutations are there of set of 5 objects?

Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.

How many different Top 10 lists can you make?

P(17,10) = 17x16x15x14x13x12x11x10x9x8

Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.

Now suppose you want to have 4 animal, 4 car, and 2 sports results in the Top 10, in that band order. How many Top 10 lists can you make?

P(6,4) x P(8,4) x P(3,2)

Permutations

Suppose your search engine returns Top 10 ranked web pages for the search query "jaguar". You have 6 top ranked pages for the animal, 8 for the car, and 3 for a sports team.

Now suppose you want to have 4 animal, 4 car, and 2 sports results in the Top 10, **the order doesn't matter**, but you want the results of the same type to appear together. How many Top 10 lists can you make?

P(6,4) x P(8,4) x P(3,2) x 3!

Permutations

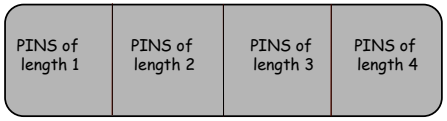
In how many ways can 5 distinct Martians and 3 distinct Jovians stand in line, if no two Jovians stand together?

___ M1 ___ M2 ___ M3 ___ M4 ___ M5 ___

5! X P(6,3)

Disjoint Sets: Addition Rule

How many PINs of length **at most 4** are there with letters, numbers, repetitions allowed, not case sensitive?



$36 + 36^2 + 36^3 + 36^4$

Suppose a finite set A equals the union of k distinct mutually disjoint subsets $A_1, A_2, \ldots, A_k$. Then

$$N(A) = N(A_1) + N(A_2) + \ldots + N(A_k) = \sum_{i=1}^{k} N(A_i)$$

Disjoint Sets: Addition Rule

- How many three digit integers are divisible by 5?
  End with 0 + end with 5

- How many times is the statement *Statement* executed?

```
for i=1 to n
    for j=1 to i
        Statement
```

Announcement

Google visit: Tuesday Sept 27, 4-7:30pm
Cardinal Room, Student Center East (SCE)

- Graduate Googler Panel: 4pm-5pm, Join us for an insightful panel featuring UIC PhD alumni as they chat about what they work on@Google, their transition from academia to industry, and answer any of your questions.
- Resume & Interview Workshop: 5pm-6pm, Get insight into what we look for in resumes and tips on the interview process, including sample questions!
- Lightning Tech Talks: 6:30pm-7:30pm, Join us for some food and an exciting set of Lightning Talks covering a few of the exciting projects our Googlers are working on!

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ZebraCup

Identify zebra from their stripe pattern Do you want to secure a chance to go on a field trip to Kenya and the life-long braking right? Here is what you need to do:
beat StripeSpotter,
the state-of-the-art algorithm for identifying zebra, and beat other fellow competitors, of course. If that sounds easy enough, we will make it more challenging for you. Here is the important dates: August 29, 2011 - Registration opens September 9, 2011 - Competition begins October 21, 2011 - Competition ends and the winner will be announced.There are two tracks in the competition: Plains track and Grevy's track. Details to be announced soon

http://bit.ly/zebracuphttp://tinyurl.com/zebracuphttp://compbio.cs.uic.edu/zebracup

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Difference Rule

How many PINs of length **exactly 4** are there with letters, numbers, repetitions allowed, not case sensitive? How many contain repeated symbols?

$$36^4 - P(36,4) =$$
$$36^4 - 36!/32! =$$
$$36^4 - 36*35*34*33$$

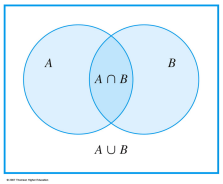
If A is a finite set and B is a subset of A then
N(A-B) = N(A) - N(B)

Difference Rule

•Python identifiers: must start with a letter (case sensitive) or _ and either end there or followed by any of those or a digit. But there are forbidden words

Inclusion/Exclusion

How many integers from 1 to 1,000 are multiples of 15?
How many multiples of either 3 or 5?



If A, B, and C are any finite sets, then

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

and

$$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$$

Combinations

A combination is an unordered selection of elements from some set.

The number of combinations of r distinct objects chosen from n distinct objects is denoted by C(n,r) or nCr or $\binom{n}{r}$, and is read "n choose r."

C(n,r) = P(n,r)/r! = n!/((n-r)!r!)

Combinations

A basketball squad consists of 12 players, 5 of which make up a team. How many different teams of players can you make from the 12?

C(12,5)

What's the diff?

In a running race of 12 sprinters, each of the top 5 finishers receives a different medal. How many ways are there to award the 5 medals?

P(12,5) = C(12,5) x 5!

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can 3 frosh and 5 soph be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **exactly** 1 frosh be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **at most** 1 frosh be selected?

Combinations

A committee of 8 students is to be selected from a class consisting of 19 frosh, and 34 soph.

In how many ways can a committee with **at least** 1 frosh be selected?

Combinations

- How many 8 bit strings with exactly three 1's?
- How many distinct words are there from MISSISSIPPI?

Suppose a collection consists of n objects of which
n_1 are of type 1 and are indistinguishable from each other
n_2 are of type 2 and are indistinguishable from each other
...
n_k are of type k and are indistinguishable from each other
and $\sum_{i=1}^{k} n_i = n$. Then the number of distinct permutations of the n objects is

$$\binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots\binom{n-n_1-n_2-\cdots-n_{k-1}}{n_k} = \frac{n!}{n_1!n_2!n_3!\cdots n_k!}$$