CS 301 Languages and Automata, UIC
Fall 2012, Assignment 5
Due: Friday, November 2, 2012 at start of discussion session

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [5 marks] This question asks you to examine the formal definitions of a TM and related concepts closely. Based on these definitions, answer the following.
   (a) A *configuration* of a Turing Machine (TM) consists of three things. What are these three things?
   (b) Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
   (c) Can the tape alphabet Γ be the same as the input alphabet Σ?
   (d) Can a Turing machine's head *ever* be in the same location in two successive steps?
   (e) Can a TM contain just a single state?
   (f) What is the difference between a decidable language and a Turing-recognizable language?

2. [8 marks] This question gets you to practice describing TM's at a semi-low level. Give an implementation-level description of a TM that decides the language

   $$L = \{x \mid x \text{ contains twice as many 0s as 1s}\}.$$

   By *implementation-level description*, we mean a description similar to Example 3.11 in the text (i.e. describe how the machine's head would move around, whether the head might mark certain tape cells, etc.... Please do *not* draw a full state diagram (for your sake and for ours)).

3. This question investigates a variant of our standard TM model from class. Our standard model included a tape which was infinite in one direction only. Consider now a TM whose tape is infinite in *both* directions (i.e. you can move left or right infinitely many spaces on the tape). We call this a TM with *doubly infinite tape*.
   (a) [3 marks] Show that a TM with doubly infinite tape can simulate a standard TM.
   (b) [5 marks] Show that a standard TM can simulate a TM with doubly infinite tape.
   (c) [1 mark] What does this imply about the sets of languages recognized by both models?

4. This question studies closure properties of the decidable and Turing-recognizable languages.
   (a) [5 marks] Show that the set of decidable languages is closed under complement.
   (b) [5 marks] Show that the set of decidable languages is closed under concatenation.
   (c) [5 marks] Show that the set of Turing-recognizable languages is closed under concatenation.

5. [6 marks] Let $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

6. This question allows you to explore variants of the computational models we've defined in class. Let a k-PDA be a pushdown automaton that has k stacks. In this sense, a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

(a) [5 marks] Show that 2-PDAs are more powerful than 1-PDAs. (Hint: It may help to use the fact that the context-free languages are a strict subset of the decidable languages.)

(b) [5 marks] Show that 3-PDAs are not more powerful than 2-PDAs. (Hint: Simulate a TM tape with two stacks.)

7. [5 marks] This question tests your comfort with concepts of countability. Let B denote the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable using a proof by diagonalization.

8. [Bonus, 6 marks] Let A be a Turing-recognizable language consisting of descriptions of TMs, $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$, where each M_i is a decider. Prove that some decidable language D is not decided by any M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)