$\binom{n}{m}$	Combinations	Subsets of size m	$n!$	Factorial	Permutations
$\left[{n \atop m}\right]$	Stirling numbers (1st)	Permutations with m cycles.	H_n	Harmonic	Sum of reciprocals to n
$\left\{{n \atop m}\right\}$	Stirling numbers (2nd)	Partitions into m non-empty sets.	C_n	Catalan	Ordered binary trees with $n+1$ leaves
$\left\langle{n \atop m}\right\rangle$	1st order Eulerian numbers	Permutations with m ascents.	F_n	Fibonacci	Sequences of 1's and 2's adding to $n-1$
$p_{n,m}$	Integer partitions	of n with largest part m.	P_n	Partitions	Integer partitions adding to n
$d_{n,m}$	Permutations	of n with exactly m fixed points.	B_n	Bell	Set partitions on an n element set
(a,b)	$\gcd(a,b)$	greatest common divisor	D_n	Derangements	Permutations with no fixed points

$$n! = n(n-1)! = \prod_{k=1}^n k \quad (0!=1)$$
$$= \binom{n}{n/2}(n/2)!^2$$

$$H_n = \sum_{k=1}^n \frac{1}{n}, H_n^{(r)} = \sum_{k=1}^n \frac{1}{n^r}$$
$$\ln n < H_n < 1 + \ln n$$
$$\sum_{k=1}^{n-1} H_k = nH_n - n$$
$$\sum_{k=1}^{n} \binom{k}{m} H_k = \binom{n+1}{m+1}\left(H_{n+1} - \frac{1}{m+1}\right)$$
$$\sum_{k=1}^{n} H_k^2 = (n+1)H_n^2 - (2n+1)H_n + 2n$$
$$\sum_{k=1}^{n} \frac{H_k}{k} = \frac{1}{2}(H_n^2 + H_n^{(2)})$$
$$H_\infty^{(2)} = \pi^2/6, H_\infty^{(4)} = \pi^4/90$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-k-1} \quad (C_0 = 1)$$
$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{1}{n}\binom{2n}{n+1} = \frac{1}{2n+1}\binom{2n+1}{n}$$
$$= \binom{2n}{n} - \binom{2n}{n-1}$$
$$= C_{n-1}\frac{2(2n-1)}{n+1}$$

$$\phi^2 - \phi - 1 = 0, \phi = \frac{1+\sqrt5}{2}, \hat\phi = \frac{1-\sqrt5}{2}$$
$$\phi - 1 = \frac{1}{\phi} = -\hat\phi$$

$$F_0 = 0, F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$
$$= \frac{\phi^n - \hat\phi^n}{\sqrt5} = \left\lfloor \frac{\phi^n}{\sqrt5} \right\rceil$$
$$= 1 + \sum_{k=0}^{n-2} F_k$$
$$= 1 + \sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor} F_{n-2k}$$
$$F_{-n} = (-1)^{n-1}F_n$$
$$F_{n+m} = F_{n+1}F_m + F_nF_{m-1}$$
$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n$$
$$F_{2n+1} = 1 + \sum_{k=0}^n F_{2k}$$
$$F_nF_{n+1} = \sum_{k=0}^n F_k^2$$
$$F_{2n}^2 = \sum_{k=1}^{2n} F_kF_{k-1}$$
$$F_{3n} = F_{n+1}^3 + F_n^3 - F_{n-1}^3$$
$$F_n^2 - F_{n+r}F_{n-r} = (-1)^{n-r}F_r^2$$
$$F_nF_{m+1} - F_{n+1}F_m = (-1)^m F_{n-m}$$
$$\sum_{k=1}^\infty \frac{(-1)^{k+1}}{F_kF_{k+1}} = \phi - 1$$
$$F_n \mid F_{mn},\ \gcd(F_n, F_m) = F_{\gcd(n,m)}$$

$$D_1 = 0, D_2 = 1$$
$$D_n = (n-1)(D_{n-1} + D_{n-2})$$
$$= n!\sum_{k=0}^n \frac{(-1)^k}{k!} = \left\lfloor \frac{n!}{e} \right\rceil$$
$$= d_{n,0}$$
$$d_{n,m} = \binom{n}{m}D_{n-m}$$
$$d_{n,n} = 1$$
$$d_{n,n-1} = 0$$
$$d_{n,n-2} = \binom{n}{2}$$
$$\sum_{k=0}^n d_{n,k} = n!$$

$$B_n = \sum_{k=0}^{n-1}\binom{n}{k}B_k \quad (B_0 = 1)$$
$$= \frac{1}{e}\sum_{k>1}\frac{k^n}{k!}$$

$$n!! = n(n-2)!!, (0!! = 1!! = 1)$$
$$n! = n!!(n-1)!!, (2n)!! = 2^n n!$$
$$(2n-1)!!/(2n)!! = \binom{2n}{n}/2^{2n}$$

$$p_{n,1} = p_{n,n} = 1$$
$$p_{n,m} = p_{n-1,m-1} + p_{n-m,m}$$
$$= \sum_{k=1}^m p_{n-m,k}$$
$$= \sum_{k=1}^m p_{n-km,m-1}$$
$$p_{2n,n} = p_{2n+k,n+k} \quad k \geq 0$$
$$P_n = \sum_{k=1}^n p_{n,k} = p_{2n,n}$$
$$= P_{n-1} + \sum_{k=0}^{\lfloor n/2\rfloor} p_{n-k,k}$$
$$= \sum_{m=1}^n \sum_{k=1}^{n-m+1} p_{m,k}$$
$$= \sum_{k\geq1} P_{n-k(3k\pm1)/2}$$
$$P_{n-m} = p_{n,m} \qquad m \geq \lfloor n/2 \rfloor$$

$$\gcd(a,0) = a$$
$$\gcd(a,b) = \gcd(a-b,b)$$
$$= \gcd(a \bmod b, b)$$
$$= a + b - ab + 2\sum_{k=1}^{b-1}\left\lfloor \frac{ka}{b} \right\rfloor$$
$$\gcd(da,db) = d\gcd(a,b)$$
$$ab = \gcd(a,b)\,\mathrm{lcm}(a,b)$$
$$n/\gcd(a,b) = \mathrm{lcm}(n/a, n/b)$$

$$\binom{n}{0} = \binom{n}{n} = 1$$
$$\left[{n \atop 0}\right] = [n=0], \left[{n \atop n}\right] = 1$$
$$\left\{{n \atop 1}\right\} = \left\{{n \atop n}\right\} = 1$$
$$\left\langle{n \atop 0}\right\rangle = \left\langle{n \atop n-1}\right\rangle = 1$$

$$\binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1}$$
$$\left[{n \atop m}\right] = (n-1)\left[{n-1 \atop m}\right] + \left[{n-1 \atop m-1}\right]$$
$$\left\{{n \atop m}\right\} = m\left\{{n-1 \atop m}\right\} + \left\{{n-1 \atop m-1}\right\}$$
$$\left\langle{n \atop m}\right\rangle = (m+1)\left\langle{n-1 \atop m}\right\rangle + (n-m)\left\langle{n-1 \atop m-1}\right\rangle$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$
$$\sum_{k=0}^n \left[{n \atop k}\right] = n!$$
$$\sum_{k=0}^n \left\{{n \atop k}\right\} = B_n$$
$$\sum_{k=0}^n \left\langle{n \atop k}\right\rangle = n!$$

$$\binom{-n}{m} = (-1)^m\binom{m-n-1}{m}$$
$$\left[{-n \atop -m}\right] = (-1)^m\left\{{n \atop m}\right\}$$
$$\left\{{-n \atop -m}\right\} = (-1)^m\left[{n \atop m}\right]$$
$$\left\langle{-n \atop -m}\right\rangle = ***$$

$$\binom{n}{m} = \binom{n}{n-m}$$
$$\left\langle{n \atop m}\right\rangle = \left\langle{n \atop n-1-m}\right\rangle$$

$$n^k/k! + \Theta(n^{k-1})$$
$$k^n/k! + ??$$

$$\binom{n}{2} = \frac{n(n-1)}{2}$$
$$\left[{n \atop 1}\right] = (n-1)!$$
$$\left\{{n \atop 2}\right\} = 2^{n-1} - 1$$
$$\left\langle{n \atop 1}\right\rangle = 2^n - n - 1$$

$$\binom{n}{n-2} = \binom{n}{2}$$
$$\left[{n \atop n-1}\right] = \binom{n}{2}$$
$$\left\{{n \atop n-1}\right\} = \binom{n}{2}$$
$$\left\langle{n \atop n-1}\right\rangle = 1$$

$$\binom{n+1}{m+1} = \sum_{k=0}^n \binom{k}{m}$$
$$\left[{n+1 \atop m+1}\right] = \sum_{k=0}^n \left[{k \atop m}\right]\frac{n!}{k!}$$
$$\left\{{n+1 \atop m+1}\right\} = \sum_{k=0}^n \left\{{k \atop m}\right\}(m+1)^{n-k}$$
$$\left\langle{n+1 \atop m+1}\right\rangle = \sum_{k=0}^n \left\langle{k \atop m}\right\rangle\frac{k-m}{(m+2)^{k-n}}$$

$$\binom{n+m+1}{m} = \sum_{k=0}^m \binom{n+k}{k}$$
$$\left[{n+m+1 \atop m}\right] = \sum_{k=0}^m \left[{n+k \atop k}\right]k(n+k)$$
$$\left\{{n+m+1 \atop m}\right\} = \sum_{k=0}^m \left\{{n+k \atop k}\right\}k$$
$$\left\langle{n+m+1 \atop m}\right\rangle = \sum_{k=0}^m \left\langle{n+k \atop k}\right\rangle\frac{k+1}{(n+1)^{m-k}}$$

$$\binom{n}{m} = \sum_{k=0}^m (-1)^{m-k}\left\{{n+1 \atop k+1}\right\}\left[{k \atop m}\right]$$
$$\left[{n \atop m}\right] = \sum_{k=0}^m (-1)^{m-k}\left[{n+1 \atop k+1}\right]\left\{{k \atop m}\right\}$$
$$\left\{{n \atop m}\right\} = \sum_{k=0}^m (-1)^{n-k}\binom{n}{k}\left\{{k+1 \atop m+1}\right\}$$
$$\left\langle{n \atop m}\right\rangle = \sum_{k=0}^m (-1)^{m-k} ***$$

$$\binom{0}{m} = \left\{{0 \atop m}\right\} = \left[{0 \atop m}\right] = \left\langle{0 \atop m}\right\rangle = [m=0]$$
$$\binom{n}{m} = \frac{n!}{(n-m)!m!} = \prod_{k=1}^m \frac{n-k+1}{k}$$
$$\binom{n}{m}[\neg(0 \leq m \leq n)] = 0$$
$$\binom{n}{m}\binom{m}{r} = \binom{n}{r}\binom{n-r}{m-r}$$
$$\sum_{k=0}^n (-1)^k\binom{n}{k} = 0$$
$$\sum_{k=0}^n k\binom{n}{k} = n2^{n-1}$$
$$\sum_{k=0}^n \binom{n}{k}\binom{k}{m} = \binom{n}{m}2^{n-m}$$
$$\sum_{k=0}^{n-1} (k+1)\left\langle{n \atop k}\right\rangle = (n+1)!/2$$
$$\sum_{k=0}^{n/2} \binom{n}{2k} = 2^{n-1}$$
$$\sum_{k=0}^r \binom{n}{k}\binom{m}{r-k} = \binom{n+m}{r}$$
$$\sum_{k=0}^n \binom{n}{s+k}\binom{m}{r+k} = \binom{n+m}{n-r+s}$$
$$\sum_{k=0}^n \binom{n-k}{s}\binom{m+k}{r} = \binom{n+m+1}{r+s+1}$$
$$\sum_{k=-s}^r \binom{n}{s+k}\binom{m}{r-k} = \binom{n+m}{r+s}$$
$$\sum_{k=0}^{2n} (-1)^k\binom{2n}{k}^3 = (-1)^n\binom{3n}{n\ n\ n}$$
$$\prod_{k=1}^n \binom{a_1+\ldots+a_k}{a_1+\ldots+a_{k-1}} = \binom{a_1+\ldots+a_n}{a_0\ldots a_n}$$

$$\left[{n \atop m}\right] \geq \left\{{n \atop m}\right\}$$
$$\left[{n+1 \atop m+1}\right] = \sum_{k=0}^n \left[{n \atop k}\right]\binom{k}{m}$$
$$\left\{{n+1 \atop m+1}\right\} = \sum_{k=0}^n \left\{{k \atop m}\right\}\binom{n}{k}$$
$$\left\{{n \atop m}\right\} = \sum_{k=1}^m (-1)^{m-k}\frac{k^n}{m!}\binom{m}{k}$$
$$\left\{{n \atop m}\right\}n^{\underline{n-k}} = n!\sum_{k=0}^n \frac{1}{k!}\left[{k \atop m}\right]$$
$$(n-m)!\binom{n}{m} = \sum_k \left[{n+1 \atop k+1}\right]\left\{{k \atop m}\right\}(-1)^{m-k}$$
$$\left\{{n \atop n-m}\right\} = \sum_k \binom{m-n}{m+k}\binom{m+n}{n+k}\left[{m+k \atop k}\right]$$
$$\left[{n \atop n-m}\right] = \sum_k \binom{m-n}{m+k}\binom{m+n}{n+k}\left\{{m+k \atop k}\right\}$$
$$\left\{{n \atop \ell+m}\right\}\binom{\ell+m}{\ell} = \sum_k \left\{{k \atop \ell}\right\}\left\{{n-k \atop m}\right\}\binom{n}{k}$$
$$\left[{n \atop \ell+m}\right]\binom{\ell+m}{\ell} = \sum_k \left[{k \atop \ell}\right]\left[{n-k \atop m}\right]\binom{n}{k}$$
$$[n=m] = \sum_{k=0}^n (-1)^{n-k}\left[{n \atop k}\right]\left\{{k \atop m}\right\}$$
$$[n=m] = \sum_{k=0}^n (-1)^{k-m}\left\{{n \atop k}\right\}\left[{k \atop m}\right]$$
$$\frac{2n}{n+1} = \sum_{k=0}^n k\binom{n}{k}\binom{2n-1}{k}^{-1}$$
$$\frac{1}{1-2n} = \sum_{k=0}^n (-1)^k\binom{n}{k}\binom{2k}{k}^{-1}2^{2k}$$
$$2 = \sum_{k=0}^n \binom{n}{k}\binom{2n-1}{k}^{-1}$$

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1} = \frac{n}{n-k}\binom{n-1}{k} = \frac{n-k+1}{k}\binom{n}{k-1}$$
$$2^n = \sum_{k\geq0}\binom{n+k}{k}2^{-k}$$
$$2^{2n} = \sum_{k=0}^n \binom{2n}{k} = \sum_{k=0}^n \binom{2n+1}{2k}$$
$$x^n = \sum_{k=0}^n \left\langle{n \atop k}\right\rangle\binom{x+k}{n}$$
$$\left\langle{n \atop m}\right\rangle = \sum_{k=0}^m \binom{n+1}{k}(m+1-k)^n(-1)^k$$
$$m!\left\{{n \atop m}\right\} = \sum_{k=0}^n \left\langle{n \atop k}\right\rangle\binom{k}{n-m}$$
$$\left\langle{n \atop m}\right\rangle = \sum_{k=0}^n \left\{{n \atop k}\right\}\binom{n-k}{m}(-1)^{n-k-m}k!$$
$$n^n(H_n - 1) = \sum_{k\geq1}\binom{n}{k}\frac{(-1)^{k-1}}{k}(n-k)^n$$
$$n! = \sum_k (-1)^k\binom{n}{k}k^n$$
$$F_{n+1} = \sum_{k\geq0}\binom{n-k}{k} = \sum_{k\geq0}\binom{k}{n-k}$$
$$F_{2n} = \sum_{k=0}^n \binom{n}{k}F_k$$
$$\binom{2n}{n} = \sum_{k=n}^{n^2} p_{k,n}$$
$$\left[{n \atop 2}\right] = (n-1)!H_{n-1}$$
$$\binom{a+b+c}{a\ b\ c} = \sum_{k=-a}^a (-1)^k\binom{a+b}{a+k}\binom{b+c}{b+k}\binom{c+a}{a+k}$$
$$\binom{n-1}{m-1}\binom{n}{m+1}\binom{n+1}{m} = \binom{n-1}{m}\binom{n}{m-1}\binom{n+1}{m+1}$$

functions $|D| = n$ (objects,slots), $|R| = k$ (boxes,values)
Distinguishable (D) vs. Indistinguishable (I), $|f^{-1}(x)|$

domain n objects	range m boxes	inject $= 0,1$	biject $=1$	surject ≥ 1	arbitrary ≥ 0
D	D	$k^{\underline{n}}$	$m!$	$m!\left\{{n \atop m}\right\}$	m^n
I	D	$\binom{m}{n}$	$[n=m]$	$\binom{n-1}{m-1}$	$\binom{n+m-1}{n}$
D	I	$[n \leq m]$	$[n=m]$	$\left\{{n \atop m}\right\}$	$\sum_{k=1}^m \left\{{n \atop k}\right\}$***
I	I	$[n \leq m]$	$[n=m]$	$p_{n,m}$	$\sum_{k=1}^m p_{n,k}$

Asymptotics

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \frac{571}{2488320n^4} + \Theta\left(n^{-5}\right)\right)$$
$$C_n = \frac{2^{2n}}{(n+1)\sqrt{\pi n}}\left(1 - \frac{1}{8n} + \frac{1}{128n^2} + \frac{5}{1024n^3} - \frac{21}{32768n^5} + \mathrm{O}\left(n^{-5}\right)\right)$$
$$P_n = \exp(\pi\sqrt{2n/3})\left(1 + \frac{1}{4n\sqrt3} + o\left(\frac{1}{n^{1/2}}\right)\right)$$
$$H_n = \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} + \mathrm{O}\left(\frac{1}{n^6}\right)$$
$$p_n = n\ln n + n\ln\ln n - n + n\frac{\ln\ln n}{\ln n} + \mathrm{O}\left(\frac{n}{\ln n}\right)$$
$$\pi(n) = \frac{n}{\ln n} + \frac{n}{(\ln n)^2} + \frac{2!n}{(\ln n)^3} + \mathrm{O}\left(\frac{n}{(\ln n)^4}\right)$$

Rising/Falling Factorial Powers:

$$x^{\underline{n}} = x(x-1)\cdots(x-n+1)$$
$$= \frac{x!}{(x-n)!} = \prod_{k=0}^{n-1}(x-k)$$
$$x^{\underline{0}} = 1, x^{\underline{x}} = x!, \binom{x}{n} = \frac{x^{\underline{n}}}{n!}$$
$$x^{\underline{-n}} = \frac{1}{(x+1)^{\overline{n}}}$$
$$x^{\underline{n+m}} = x^{\underline{m}}(x-m)^{\underline{n}}$$

$$x^{\overline{n}} = x(x+1)\cdots(x+n-1)$$
$$= \frac{(x+n-1)!}{(x-1)!} = \prod_{k=0}^{n-1}(x+k)$$
$$x^{\overline{0}} = 1, 1^{\overline{x}} = x!, \binom{x+n-1}{n} = \frac{x^{\overline{n}}}{n!}$$
$$x^{\overline{-n}} = \frac{1}{(x-1)^{\underline{n}}}$$
$$x^{\overline{n+m}} = x^{\overline{m}}(x+m)^{\overline{n}}$$

$$x^{\underline{n}} = (-1)^n(-x)^{\overline{n}}$$
$$= (x-n+1)^{\overline{n}} = 1/(x+1)^{\overline{-n}}$$
$$x^{\underline{n}} = \sum_{k=1}^n \left[{n \atop k}\right](-1)^{n-k}x^k$$
$$x^n = \sum_{k=1}^n \left\{{n \atop k}\right\}x^{\underline{k}}$$

$$x^{\overline{n}} = (-1)^n(-x)^{\underline{n}}$$
$$= (x+n-1)^{\underline{n}} = 1/(x-1)^{\underline{-n}}$$
$$x^{\overline{n}} = \sum_{k=1}^n \left[{n \atop k}\right]x^k$$
$$x^n = \sum_{k=1}^n \left\{{n \atop k}\right\}(-1)^{n-k}x^{\overline{k}}$$

Finite Difference Operators: $Ef(x) = f(x+1)$

$\Delta f(x) = (E-1)f(x)$

$$\Delta F(x) = f(x)$$
$$\Delta(cu) = c\Delta u$$
$$\Delta(u+v) = \Delta u + \Delta v$$
$$\Delta(uv) = u\Delta v + Ev\Delta u$$
$$\Delta(x^{\underline{n}}) = nx^{\underline{n-1}}$$
$$\Delta(H_x) = x^{\underline{-1}}$$
$$\Delta(2^x) = 2^x$$
$$\Delta(c^x) = (c-1)c^x$$
$$\Delta\binom{x}{m} = \binom{x}{m-1}$$

$\sum_a^b f(x)\delta x = \sum_{i=a}^{b-1} f(i)$

$$\sum f(x)\,\delta x = F(x) + C$$
$$\sum cu\,\delta x = c\sum u\,\delta x$$
$$\sum(u+v)\,\delta x = \sum u\,\delta x + \sum v\,\delta x$$
$$\sum u\Delta v\,\delta x = uv - \sum Ev\Delta u\,\delta x$$
$$\sum x^{\underline{n}}\,\delta x = \frac{x^{\underline{n+1}}}{n+1}$$
$$\sum x^{\underline{-1}}\,\delta x = H_x$$
$$\sum 2^x\,\delta x = 2^x$$
$$\sum c^x\,\delta x = \frac{c^x}{c-1}$$
$$\sum \binom{x}{m}\,\delta x = \binom{x}{m+1}$$

GF	form	$[x^k](A(x)\cdot B(x))$
ordinary	$\sum a_k x^k$	$\sum_{j=0}^{k} a_j b_{k-j}$
		$\sum_{r+s=k} a_r b_s$
exponential	$\sum a_k \frac{x^k}{k!}$	$\sum_j \binom{k}{j} a_j b_{k-j}$
		$\sum_{r+s=k} \binom{k}{r} a_r b_s$
Newtonian	$\sum a_k \binom{x}{k}$	$\sum_{i,j} \binom{k}{i}\binom{k-i}{j} a_{i+j} b_{k-j}$
		$\sum_{r+s+t=k} \binom{k}{r\ s\ t} a_{r+s} b_{s+t}$
Dirichlet	$\sum \frac{a_k}{k^x}$	$\sum_{d\mid k} a_d b_{\frac{k}{d}}$
		$\sum_{r\cdot s=k} a_r b_s$

ogf	coeffs	func
$\frac{1}{1-x}$	$1\,1\,1\,1\cdots$	1
$\frac{1}{1-cx}$	$1\,c\,c^2\,c^3\cdots$	c^k
$\frac{x^{n+1}-1}{x-1}$	$1\,1\cdots1\,0\cdots$	$1, k\le n$
$\frac{1}{1-x^n}$	$1\,0\cdots1\,0\cdots$	$[n\mid k]$
$\frac{x}{(1-x)^2}$	$0\,1\,2\,3\,4\cdots$	k
$\left(x\frac{d}{dx}\right)^n\left(\frac{1}{1-x}\right)$	$0^n\,1^n\,2^n\,3^n\,4^n\cdots$	k^n
e^x	$1\,1\,\frac12\,\frac16\,\frac1{24}\cdots$	$\frac{1}{k!}$
$\ln\frac{1}{1-x}$	$0\,1\,\frac12\,\frac13\,\frac14,\cdots$	$\frac1k$
$\frac{1}{\sqrt{1-4x}}$	$1\,1\,2\,6\,20\cdots$	$\binom{2k}{k}$

$\frac{x}{1-qx-x^2}$	$F_k(q)$
$\frac{1}{2x}(1-\sqrt{1-4x})$	C_k
$\frac{1}{1-x}\log\frac{1}{1-x}$	H_k
e^{e^x-1}	$\frac{B_k}{k!}$
$\prod_{k\ge1}\frac{1}{1-x^k}$	P_k
$\frac{e^{-x}}{1-x}$	D_k
$x\frac{e^{qx}}{e^x-1}$	$\mathcal{B}_k(q)$
$\frac{2e^{qx}}{e^x+1}$	$\mathcal{E}_k(q)$
$\frac{1}{(1-x)^{n+1}}\ln\frac{1}{1-x}$	$(H_{n+k}-H_n)\binom{n+k}{k}$
$\prod_i\frac{1}{1-x_i}$	$\binom{\sum_i x_i}{x_1\cdots x_n}$

Operations: $[x^n]A(x)=a_n$

$[x^n]x^kA(x)$	$=$	a_{n-k}
$[x^n]A(cx)$	$=$	c^na_n
$[x^n]A'(x)$	$=$	$(n+1)a_n$
$[x^n]\int A(x)\,dx$	$=$	$\frac{a_{n-1}}{n}$
$[x^n]\frac{1}{1-x}A(x)$	$=$	$\sum^n a_i$
$[x^n/n!]e^xA(x)$	$=$	$1/n!\sum^n a_i$
$[x^{2n}]\frac{A(x)+A(-x)}{2}$	$=$	a_{2n}
$[x^{kn}]\frac1k\sum_{j=0}^{k-1}A(xe^{\frac{2\pi ji}{k}})$	$=$	a_{kn}

Sums of Powers:

$\sum_{k=1}^{n-1}k$	$=$	$\frac{n(n-1)}{2}=\binom n2$
$\sum_{k=1}^{n-1}k^2$	$=$	$\frac{(n-1)n(2n-1)}{6}=2\binom n3+\binom n2$
$\sum_{k=1}^{n-1}k^3$	$=$	$\binom n2^2=6\binom n4+6\binom n3+\binom n2$
$\sum_{k=1}^{n-1}k^m$	$=$	$\sum_{k=1}^m k!\left\{{m\atop k}\right\}\binom{n}{k+1}$
	$=$	$\frac{1}{m+1}\sum_{k=0}^m\binom{m+1}{k}\mathcal{B}_kn^{m+1-k}$
$\sum_{k\ge1}k^mz^k$	$=$	$\frac{\sum_{j=0}^m\left\langle{k\atop j}\right\rangle z^j}{(1-z)m+1}$

x^n-y^n	$=$	$(x-y)\sum_{k=0}^{n-1}x^{n-1-k}y^k$
$\prod_{k\ge0}\frac{1}{1-x^{2k+1}}$	$=$	$\prod_{k\ge0}(1+x^k)$
$\prod_{k\ge0}(1+x^{2k+1})$	$=$	$\sum_{k\ge0}\frac{x^{k^2}}{\prod_{j=1}^k(1-x^{2j})}$
$\prod_{k\ge0}(1+x^{2k})$	$=$	$\sum_{k\ge0}\frac{x^{k(k+1)}}{\prod_{j=1}^k(1-x^{2j})}$
$\frac{1}{1-x}$	$=$	$\prod_{k\ge0}\frac{1}{1+x^{2^k}}$
$F(x)=\frac{1}{1-x(1+x)}$	$=$	$x+(x+x^2)F(x)$
$C(x)=\frac{1}{1-xC(x)}$	$=$	$1+xC(x)^2$

$(x+y)^n$	$=$	$\sum_{k\ge0}\binom nk x^ky^{n-k}$
$x^{\overline{n}}$	$=$	$\sum_{k\ge0}\left[{n\atop k}\right]x^k$
$\left(\frac1x\right)^{\overline{-n}}$	$=$	$\sum_{k>0}\left\{{k\atop n}\right\}x^k$

$\frac{x^n}{(1-x)^{n+1}}$	$=$	$\sum_{k\ge n}\binom kn x^k$
$(\ln\frac{1}{1-x})^n$	$=$	$n!\sum_{k\ge0}\left[{k\atop n}\right]\frac{x^k}{k!}$
$(e^x-1)^n$	$=$	$n!\sum_{k\ge0}\left\{{k\atop n}\right\}\frac{x^k}{k!}$
$(1+x)^{-n}$	$=$	$\sum_{k>0}\binom{n+k-1}{k}x^k$

e^{x+xy}	$=$	$\sum_{n,k\ge0}\binom nk\frac{x^ny^k}{n!}$
$\frac{1}{(1-x)^y}$	$=$	$\sum_{n,k\ge0}\left[{k\atop n}\right]\frac{x^ky^n}{k!}$
$e^{y(e^x-1)}$	$=$	$\sum_{n,k\ge0}\left\{{k\atop n}\right\}\frac{x^ky^n}{k!}$
$\frac{1-y}{e^{(y-1)x}-y}$	$=$	$\sum_{n,k\ge0}\left\langle{n\atop k}\right\rangle\frac{x^ny^k}{n!}$
$\frac{e^{x(y-1)}}{1-x}$	$=$	$\sum_{n,k>0}d_{n,k}\frac{x^ny^k}{n!}$

$[x^n]f(x)h(x)$	$=$	$\sum_{k=0}^n h_{n-k}f_k$
$[x^n]f(x)^t$	$=$	$\sum_{n=\sum_{i=1}^t k_i}\prod_{i=1}^t f_{k_i}$
$[x^n]\frac{1}{1-f(x)}$	$=$	$\sum_{k=1}^n g_{n-k}f_k$
$[x^n]e^{f(x)}$	$=$	$1/n\sum_{k=1}^n kg_{n-k}f_k$
$[x^n]\ln\frac{1}{1-f(x)}$	$=$	$f_n+1/n\sum_{k=1}^n kg_kf_{n-k}$

Lambert/PowerLog:

$x=W(x)e^{W(x)},\log x=W(x)\log W(x)$

$-W(-x)=T(x)=\sum_{n>1}n^{n-1}\frac{x^n}{n!}$

Cauchy Integration:

$[x^n]f(x)=\frac{1}{2\pi i}\oint\frac{f(x)dx}{x^{n+1}}$

Lagrange Inversion: $A(x)=f^{\langle-1\rangle}(x)$

$R(x)=x/f(x)\to A(x)=xR(A(x))$

$A(x)=\sum_{n\ge1}\left(\frac{d}{dt}\right)^{n-1}R(t)^n\Big|_{t=0}\frac{x^n}{n!}$

$n[x^n]g(A(x))=[t^n]tg'(t)R^n(t)$

$[x^n]\frac{g(A(x))}{1-xR'(x)}=[t^n]g(t)R^n(t)$

$[x^n]g(A(x))=[t^n](1-t\frac{R'(t)}{R(t)})g(t)R^n(t)$

Group G_n	Cycle Index $Z(G)$	$\sum G_n$	$Z(\sum G)$
Identity I_n	$A(x)^n$	Sequence	$\frac{1}{1-A(x)}$
Cyclic C_n	$\frac1n\sum_{d\mid n}\phi(d)A(x^d)^{n/d}$	Cycle	$\sum_{n\ge1}\frac{\phi(d)}{d}\log\frac{1}{1-A(x^d)}$
Dihedral D_n	$\frac12Z(C_n)+$	Necklace	$\frac12\mathrm{Cyc}(A(x))+$
even	$\frac14(A(x)^2A(x^2)^{\frac{n-2}{2}}+A(x^2)^{\frac n2})$		$\frac{\mathrm{Seq}(A(x^2))}{4}\times$
odd	$\frac12A(x)A(x^2)^{\frac{n-1}{2}}$		$(2A(x)+A(x^2)+A(x)^2)$
Symmetric S_n	$\sum_{e\in\lambda(n)}\prod\frac{x_k^{e_k}}{k^{e_k}e_k!}$	Multiset	$\exp(\sum_{k\ge1}A(x^k)/k)$
One-To-One S'_n	$\sum_{e\in\lambda(n)}(-1)^{e_2+e_4\cdots}\prod\frac{x_k^{e_k}}{k^{e_k}e_k!}$	Set	$\exp(\sum_{k\ge1}(-1)^{k+1}\frac{A(x^k)}{k})$

Probability	
$p(x)$	$\Pr[a=X<x]$
PDF	$\Pr[a\le X<b]=\int_a^b p(x)\,dx,\sum_a^{b-1}p(x)$
$\mathrm{E}[g(X)]$	$\int_{-\infty}^{\infty}g(x)\cdot p(x)\,dx,\sum_x g(x)\Pr[X=x]$
$\Pr[X\vee Y]=$	$\Pr[X]+\Pr[Y]-\Pr[X\wedge Y]$
$\Pr[X\wedge Y]=$	$\Pr[X]\cdot\Pr[Y]$, iff X and Y are indep.
$\Pr[X\mid Y]=$	$\frac{\Pr[X\wedge Y]}{\Pr[Y]}$
$\Pr[B_i\mid A]=$	$\frac{\Pr[A\mid B_i]\Pr[B_i]}{\Pr[A](=\sum_{k=1}^n\Pr[B_k]\Pr[A\mid B_k])}$, (Bayes)
$\Pr[\bigcup_{i=1}^nX_i]=$	$\sum_{k=1}^n(-1)^{k+1}\sum_{i_i<\cdots<i_k}\Pr\left[\bigcap_{j=1}^kX_{i_j}\right]$
$\mathrm{E}[X\cdot Y]=$	$\mathrm{E}[X]\cdot\mathrm{E}[Y]$, iff X and Y are indep.
$\mathrm{E}[aX+Y]=$	$a\mathrm{E}[X]+\mathrm{E}[Y]$, (linearity of expectation)
$\mathrm{Var}[X]=$	$\sigma^2=\mathrm{E}[(X-\mathrm{E}[X])^2]=\mathrm{E}[X^2]-\mathrm{E}[X]^2$
$\mathrm{Var}[aX\pm Y]=$	$\mathrm{Var}[X]+c^2\mathrm{Var}[Y]\pm2(E_{XY}-E_XE_Y)$
Inequalities :	
Cauchy	$\mathrm{E}[\lvert XY\rvert]^2\le\mathrm{E}[X^2]\,\mathrm{E}[Y^2]$
Jensen	$f(\mathrm{E}[X])\le\mathrm{E}[f(X)]$, f convex
Markov	$\Pr[\lvert X\rvert\ge\lambda]\le\frac{\mathrm{E}[f(\lvert X\rvert)]}{f(\lambda)}$, $f>0$ monotonic
Chebyshev	$\Pr[\lvert X-\mathrm{E}[X]\rvert>\lambda]\le\frac{\mathrm{Var}[X]}{\lambda^2}$
2nd Moment	$\Pr[X=0]\le\frac{\mathrm{E}[X^2]-\mathrm{E}[X]^2}{\mathrm{E}[X]^2}$
Bonferroni	$f(a,k)=\sum_{r=k}^a(-1)^{r-k}\binom rkS_r(X)$,
	$f(k+2q-1,k)\le\Pr[X=k]\le f(k+2q,k)$
Azuma	$\Pr[\lvert f(X)-\mathrm{E}[f(X)]\rvert\ge t]\le2e^{-2t^2/\sum c_i^2}$
	$\lvert f(X_i)-f(X_i')\rvert\le c_i$
Kraft	$\sum_{x\in S}\frac{1}{2^{\lvert x\rvert}}\le1$, S a prefix code
LYM	$\sum_{k=1}^n\frac{p_k}{\binom nk}\le1$

$M_X(z)$	$=$	$\mathrm{E}[e^{zX}]$
$\varphi_X(z)$	$=$	$M_X(iz)=\mathrm{E}[e^{izX}]$
$[z^n]\varphi_X(z)$	$=$	$\mathrm{E}[X^n]$
$\varphi_{aX+Y+b}(z)$	$=$	$e^{bz}\varphi_X(az)\varphi_Y(z)$

distribution	PDF	mean	variance	MGF $=M(z)$
Continuous : $x\in(-\infty,\infty),x\in\mathbf{R}$				
uni(a,b)	$\frac{1}{b-a},a\le x\le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bz}-e^{az}}{z(b-a)}$
normal(μ,σ)	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$e^{\frac{z\mu+z^2\sigma^2}{2}}$
gamma(α,β)	$\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha}$	$\beta\alpha$	$\beta^2\alpha$	$(1-\beta z)^{-\alpha}$
beta(α,β)	$\frac{\Gamma(\alpha+\beta)x^{\alpha-1}(1-x)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$F\left({\alpha\atop\alpha+\beta}\middle\vert z\right)$
exp(λ)	$\lambda e^{-\lambda x}$	$\frac1\lambda$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-z}$
$t(n)$	$\frac{\sqrt{\frac{n}{n+x^2}}^{\,n+1}}{\sqrt nB(\frac n2,\frac12)}$	0	$\frac{n}{n-2}$	!!
$F(n,m)$	$\frac{n^{\frac n2}m^{\frac m2}x^{\frac n2-1}}{(m+nx)^{\frac{n+m}{2}}B(\frac n2,\frac m2)}$	$\frac{m}{m-2}$	$2m^2(n+m-2)$	$F\left({\frac n2\atop1-\frac m2}\middle\vert\frac{zm}{-n}\right)$
Discrete : $x\in[0\ldots k],x,k\in\mathbf{N}$				
uniform	$\frac1k$	$\lfloor\frac k2\rfloor$	$\frac{k^2-1}{12}$	$\frac{z^{k+1}-1}{k(z-1)}$
binomial	$\binom nxp^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^z)^n$
neg binomial	$\binom{n+x-1}{x}p^n(1-p)^x$	$\frac{n(1-p)}{p}$	$\frac{n(1-p)}{p^2}$	$\frac{p^n}{(1-(1-p)e^z)^n}$
geometric	$p(1-p)^x$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{(1-(1-p)e^z)}$
hypergeom	$\frac{\binom{Np}{x}\binom{N(1-p)}{n-x}}{\binom Nn}$	np	$\frac{np(1-p)(N-n)}{N-1}$	!!
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ	$e^{\lambda(e^z-1)}$

Asymptotics:

$f(n)$	$\exists c>0, n_0>0, \forall n \ge n_0$	
$o(g(n))$	$\lim_{n\to\infty}\frac{f(n)}{g(n)}=0$	$f \prec g$
$O(g(n))$	$0 \le f(n) \le cg(n)$	$f \preceq g$
$\Theta(g(n))$	$O(g(n)) \wedge \Omega(g(n))$	$f \asymp g$
$\Omega(g(n))$	$0 \le cg(n) \ge f(n)$	$f \succeq g$
$\omega(g(n))$	$\lim_{n\to\infty}\frac{g(n)}{f(n)}=0$	$f \succ g$

$\widetilde{O}(g(n)) = 0 \le c\log^k n g(n) \le f(n)$

Master method:

For a recurrence $T(n) = aT(n/b) + f(n)$

$a \ge 1, b > 1, d = \log_b a$:

$f(n)$	$T(n)$
$\mathrm{O}\left(n^{d-\epsilon}\right)$	$\Theta\left(n^d\right)$
$\Theta\left(n^d\right)$	$\Theta\left(n^d \log_b n\right)$
$\Omega\left(n^{d+\epsilon}\right)$	$\Theta\left(f(n)\right)$

Linear nonhomogeneous rec rels

$a_n = c_1 a_{n-1} \ldots + c_k a_{n-k} + f(n)$

Find rootsr of $(r^k - c_1 r^{k-1} \ldots - c_k)E(f)(r)$

$E(b^n(p(x,d))) = (r-b)^d$

Solve $a_i = (\alpha_{11} + \alpha_{12} n \ldots + \alpha_{1m_{r_1}}) r_1^n \ldots + (\ldots) r_2^n$ for $i = 0, k+d$

$\log^k n \prec n^{1/k} \prec n \prec n^{\log n} \prec 2^n \prec n! \prec n^n$

Generalized binomial/exponential

$\mathcal{B}_t(z) = \sum_{k\ge 0} (tk)^{\underline{k-1}} \frac{z^k}{k!}$ $\quad$ $\mathcal{E}_t(z) = \sum_{k\ge 0}(tk+1)^{k-1}\frac{z^k}{k!}$

$\mathcal{B}_0(z) = 1+z$ $\quad$ $\mathcal{E}_0(z) = e^z$

$\mathcal{B}_1(z) = (1-z)^{-1}$ $\quad$ $\mathcal{E}_1(z) = e^{z\mathcal{E}_1(z)}$

$\mathcal{B}_2(z) = \frac{1-\sqrt{1-4z}}{2z}$

$\mathcal{B}_t(z)^{1-t} - \mathcal{B}_t(z)^{-t} = z$ $\quad$ $\mathcal{E}_t(z)^{-t}\ln\mathcal{E}_t(z) = z$

Derived Identities

$$\sum_k \binom{tk+r}{k}\binom{tn-tk+s}{n-k}\frac{r}{tk+r} = \binom{tn+r+s}{n}$$

$$\sum_k \binom{tk+r}{k}\binom{tn-tk+s}{n-k}\frac{r}{tk+r}\cdot\frac{s}{tn-tk+s} = \binom{tn+r+s}{n}\frac{r+s}{tn+r+s}$$

$$\sum_k (tk+r)^k (tn-tk+s)^{n-k}\frac{r}{tk+r} = (tn+r+s)^n$$

$$\sum_k (tk+r)^k (tn-tk+s)^{n-k}\frac{r}{tk+r}\cdot\frac{s}{tn-tk+s} = (tn+r+s)^n\frac{r+s}{tn+r+s}$$

Hypergeometric functions:

$$F\left(\begin{matrix}a_1,\cdots,a_m\\ b_1,\cdots,b_n\end{matrix}\middle|\, z\right) = \sum_{k\ge 0}\frac{a_1^{\overline{k}}\cdots a_m^{\overline{k}}}{b_1^{\overline{k}}\cdots b_n^{\overline{k}}}\frac{z^k}{k!}$$

$e^z = \sum_{k\ge 0}\frac{z^k}{k!} = F\left(\begin{matrix}1\\1\end{matrix}\middle|\, z\right)$

$\frac{1}{1-z} = \sum_{k\ge 0} z^k = F\left(\begin{matrix}1,1\\1\end{matrix}\middle|\, z\right)$

$\frac{1}{(1-z)^a} = \sum_k \binom{a+k-1}{k} z^k = \sum_{k\ge 0}\frac{a^{\overline{k}} z^k}{k!} = F\left(\begin{matrix}a,1\\1\end{matrix}\middle|\, z\right)$

$(1+z)^a = \sum_{k\ge 0}\binom{a}{k}z^k = F\left(\begin{matrix}-a,1\\1\end{matrix}\middle|\, z\right)$

$\ln(1+z) = \sum_{k\ge 1}(-1)^k z^k/k = zF\left(\begin{matrix}1,1\\2\end{matrix}\middle|\, -z\right)$

$\frac{r+n+1}{r+1} = F\left(\begin{matrix}1,-n\\-n-r\end{matrix}\middle|\, 1\right)$

$\cos(z) = \sum_{k\ge 0}(-1)^k\frac{z^{2k}}{2k!} = F\left(\begin{matrix}1\\\frac{1}{2},1\end{matrix}\middle|\, -\frac{z^2}{4}\right)$

$F\left(\begin{matrix}a,b\\c\end{matrix}\middle|\, 1\right) = \frac{\Gamma(c-a-b)\Gamma(c)}{\Gamma(c-a)\Gamma(c-b)}$

$F\left(\begin{matrix}a,-n\\c\end{matrix}\middle|\, 1\right) = \frac{(c-a)^{\overline{n}}}{c^{\overline{n}}} = \frac{(a-c)^{\underline{n}}}{(-c)^{\underline{n}}}$

Gamma, Binomial

$$\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt = \int_0^1 \log^{z-1}\frac{1}{t}dt = \lim_{n\to\infty}\frac{n^{z-1}}{\binom{n+z-1}{n}} \approx e^{-z}z^{z-1/2}\sqrt{2\pi}$$

$\Gamma(n+1) = n\Gamma(n) = n!$

$\frac{\pi}{\sin(\pi z)} = \Gamma(z)\Gamma(1-z)$

$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

$$\binom{n+m}{n} \approx \left(1+\frac{n}{m}\right)^m\left(1+\frac{m}{n}\right)^n\sqrt{\frac{1}{2\pi}\left(\frac{1}{n}+\frac{1}{m}\right)}$$

$\gamma = \lim_{n\to\infty}(H_n - \log n) = \int_0^\infty -e^{-x}\log x\,dx = -(\log\Gamma(x))'|_{x=0}$

$\prod_{n\ge 1}\frac{f(n)}{g(n)} = \prod_{j=1}^k\frac{\Gamma(1-f_j)}{\Gamma(1-g_j)}$

$\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}}\Gamma(z)\Gamma(z+\frac{1}{2})$

$\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$

$\binom{n-\frac{1}{2}}{n} = \frac{1}{2^{2n}}\binom{2n}{n} \approx \frac{1}{\sqrt{\pi n}}$

$\binom{n}{\frac{1}{2}} = \frac{2^{2n+1}}{\pi\binom{2n}{n}} \approx 2\sqrt{\frac{n}{\pi}}$

$\binom{n}{\frac{n}{2}} = \frac{2^{2n}}{\pi\binom{n}{\frac{n-1}{2}}} \approx 2^n\sqrt{\frac{n}{2\pi}}$

$\binom{-\frac{1}{2}}{n} = (-1)^n\frac{1}{2^{2n}}\binom{2n}{n}$

$\binom{-\frac{1}{2}}{n}\binom{-\frac{3}{2}}{n} = (2n+1)\binom{-\frac{1}{2}}{n}^2 = \sum_{k=0}^n\binom{\frac{1}{2}}{n}\binom{-\frac{1}{2}}{n}$

$(-1)^n\pi = (-n-\frac{1}{2})!(n-\frac{1}{2})!$

$\Gamma(z)\zeta(z) = \int_{t=0}^\infty\frac{t^{z-1}}{e^t-1}$

$\Gamma(\frac{1}{2}) = (-\frac{1}{2})! = \sqrt{\pi}$

Multiplicative functions:

f	description	$f(p^r)$	worst	avg
$\phi(n)$	$\sum_{\gcd(d,n)=1} 1$	$p^r - p^{r-1}$	$\frac{n}{\log\log n}$	$\frac{6n}{\pi^2}$
$\sigma(n)$	$\sum_{d\mid n} d$	$\frac{p^{r+1}-1}{p-1}$	$n\log\log n$	$\frac{\pi^2 n}{6}$
$\tau(n)$	$\sum_{d\mid n} 1$	$r+1$	$2^{\frac{\log n}{\log\log n}}$	$\log n$
$\mu(n)$	$(-1)^k$ for k distinct prime divisors, 0 if not square–free	$-[r=1]$	1	0
$\lvert\mu(n)\rvert$	1 if square free, 0 otherwise	$[r=1]$	1	$\frac{6}{\pi^2}$

Identities :

$g(n) = \sum_{d\mid n} f(d)$	$f(n) = \sum_{d\mid n}\mu(\frac{n}{d})g(d)$
$n = \sum_{d\mid n}\phi(d)$	$\phi(n) = \sum_{d\mid n}\mu(\frac{n}{d})d$
$\sigma_k(n) = \sum_{d\mid n} d^k$	$n^k = \sum_{d\mid n}\mu(\frac{n}{d})\sigma_k(d)$
$\tau(n) = \sum_{d\mid n} 1$	$1 = \sum_{d\mid n}\mu(\frac{n}{d})\tau(d)$

$[n=1] = \sum_{d\mid n}\mu(d)$

$g(n) = \sum_{d\prec n} f(d)$	$f(n) = \sum_{d\prec n}\mu_{1,n-d}g(d)$; $\zeta_{x,y} = [x \prec y], \mu = (\zeta + I)^{-1}$
$g(n) = \sum_{d=0}^{n-1} f(d)$	$f(n) = g(n) - g(n-1)$
$g(S) = \sum_{T\subseteq S} f(T)$	$f(S) = \sum_{T\subseteq S}(-1)^{\lvert T\rvert}g(T)$

$\sum_{\gcd(x,n)=1}\cos 2\pi x/n = \mu(n)$

Zeta (dirichlet gfs):

$\zeta(x) = \sum_{k\ge 1}\frac{1}{k^x} = \prod_{p\ge 0}\frac{1}{1-p^{-x}} = \frac{1}{\Gamma(x)}\int_0^\infty\frac{t^{x-1}e^t}{1-e^t}dt$

$\zeta(x-k)$	n^k
$\frac{\zeta(x-1)}{\zeta(x)}$	$\phi(k)$
$\zeta(x)\zeta(x-i)$	$\sigma_i(k)$
$\zeta^2(x)$	$\tau(k)$
$\frac{1}{\zeta(x)}$	$\mu(k)$
$\frac{\zeta(x)}{\zeta(2x)}$	$\lvert\mu(k)\rvert$
$f(x)\zeta(x)$	$\sum_{k>1}\sum_{d\mid k} f_d$

name	path type	funequiv	gf	Closed Form
Motzkin	N, E, NE	$M_x = 1 + xM_x + x^2M_x^2$	$= \frac{1-x-\sqrt{1-2x-3x^2}}{2x^2}$	$M(n) = -1/2\sum_{k=0}^{n+2}(-3)^k\binom{1/2}{k}\binom{1/2}{n+2-k}$
Schröder	N, k E	$S_x = 1 + xS_x + xS_x^2$	$= \frac{1-x-\sqrt{1-6x+x^2}}{2x}$	$S(n) = 2\sum_{k=0}^{n-2}\binom{2n-k-2}{n-1}\binom{n-2}{k}/n$
Narayana	N,E, m turns	$N_{x,y} = xy + (xy+x-1)N_{x,y} + xN_{x,y}^2$	$= \frac{2-x-xy-\sqrt{-4x^2y+(-2+x+xy)^2}}{2x}$	$N(n,m) = \frac{1}{n}\binom{n}{m}\binom{n}{m-1}$

Special functions	$a_1y'' + a_2y' + a_3y = 0$	$a_1f_{n+1} = a_2f_n - a_3f_{n-1}$	gfz^nx^k	$\sum_k g_{n,k}x^k$	hypergeometric
Hermite $H_n(x)$	$1, 0, -2n$	$1, 2x, 2n$	$e^{zx-z^2/2}$	$(-1)^k\binom{n}{2k}(2x)^{n-2k}$	$(2x)^nF\left(\begin{matrix}-\frac{n}{2},\frac{1-n}{2}\\ -\end{matrix}\middle|\, -\frac{1}{x^2}\right)$
Chebyshev $T_n(x)$	$1-x^2, nx, -n$	$1, 2x, 1$	$\frac{1-xz}{1-2xz+z^2}$	$\frac{n}{2}(-1)^k\binom{n-k-1}{k+1}(2x)^{n-2k}$	$F\left(\begin{matrix}-n,n\\1/2\end{matrix}\middle|\,\frac{1-x}{2}\right)$
Legendre $P_n(x)$	$1-x^2, 2x, -n$	$n+1, (2n+1)x, n$	$\frac{1}{\sqrt{1-2xz+z^2}}$	$\frac{1}{2^n}(-1)^k\binom{n}{k}\binom{2n-2k}{n}x^{n-2k}$	$F\left(\begin{matrix}-n,n+1\\1\end{matrix}\middle|\,\frac{1-x}{2}\right)$
Laguerre $L_n(x)$	$x, 1-x, n$	$n+1, 2n+1-x, n$	$\frac{e^{\frac{zx}{z-1}}}{(1-z)}$	$\binom{n}{n-k}(-1)^k/k!x^k$	$F\left(\begin{matrix}-n\\1\end{matrix}\middle|\, x\right)$
Bessel $B_{I,J}(n,x)$	$x^2, x, (x^2-n^2)$	$1, 2n/x, -1$	$e^{x/2(z-\frac{1}{z})}$		$\frac{x^n}{2^n n!}F\left(\begin{matrix}-\\n+1\end{matrix}\middle|\,\pm\frac{-x^2}{4}\right)$

Designs: (b,v,r,k,λ) $bk = vr, r(k-1) = \lambda(v-1)$, symmetric (v,k,λ): $b=v, k=r$, Steiner triple $\mathrm{STS}(v) = SYM(v,3,1)$ $\mathrm{FPP}(n)$ finite projective plane: $\mathrm{SYM}(n^2+n+1, n+1, 1)$

$\sum_{k=0}^n\binom{n}{k}(x-k)^{n-k-1}(y+k)^k = (x+y+n)^n/x$

$\sum_{k=0}^n\binom{n}{k}xy(x-ak)^{k-1}(y-a(n-k))^{n-k-1} = (x+y)(x+y-na)^{n-1}/x$

$\sum_{k=0}^n\binom{n}{k}(x-ak)^{k-1}(y-a(n-k))^{n-k} = (x+y-na)^n/x$

$\sum_{k=0}^n\binom{n}{k}(x-ak)^{k-1}(y+ak)^{n-k} = (x+y)^n/x$

$\sum_{k=0}^n\binom{n+k}{k}[x^{n+1}(1-x)^k + (1-x)^{n+1}x^k] = 1$

$\binom{n-q}{r-1} = \sum_{k=1}^r\frac{p-q}{pr-q}\binom{pr-q}{r-1}\binom{n-pk}{r-k}$

$\sum_{k=0}^n 1/\binom{n}{k} = \frac{n+1}{2^n}\sum_{k=0}^n\frac{2^k}{k+1}$

$\sum_{k\ge m}1/\binom{n+k}{k} = \frac{n}{(m-1)\binom{m+n-1}{n-1}}$

$\sum_{k\ge 0}k!/(n+k)! = \frac{1}{(n-1)(n-1)!}$

$\sum_{k=0}^n(-1)^k\binom{n}{k}\frac{1}{m+k+1} = \frac{1}{(n+m+1)\binom{n+m}{m}}$

$\sum_{k=0}^n\frac{1}{2k}\binom{n+1}{2k} = \sum_{k=0}^n\frac{2^k-1}{k+1}$

$\sum_{k=0}^n(-1)^{k-n}\binom{n}{k}\binom{n+k}{m} = \binom{n}{m-n}$

$\sum_{k=0}^n\binom{n}{k}^3 = \sum_{k=0}^n\binom{n}{k}^2\binom{2k}{n}$

$\sum_{k=0}^n\binom{n}{k}^2\binom{n+k}{k}^2 = \sum_{k=0}^n\sum_{j=0}^n\binom{n}{k}\binom{n+k}{k}\binom{k}{j}^3$

$\sum_{k=-\infty}^{\infty}\binom{a}{m-k}\binom{b}{n-k}\binom{a+b+k}{k}^2 = \binom{a+n}{m}\binom{b+m}{n}$

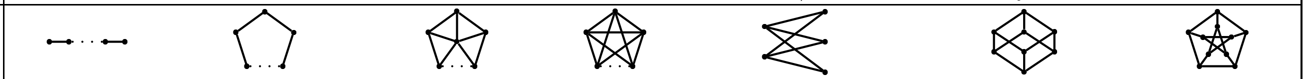
$\sum_{k=-\infty}^{\infty}\binom{a+b+c+d+e-k}{e-k}\binom{a+d}{k+d}\binom{b+c}{k+c}^2 = \binom{a+c+d+e}{a+c}\binom{b+c+d+e}{c+e}$

$x^n + y^n = \sum_{j=0}^{\lfloor n/2\rfloor}(-1)^k\frac{n}{n-k}\binom{n-k}{k}(xy)^k(x+y)^{n-2k}$

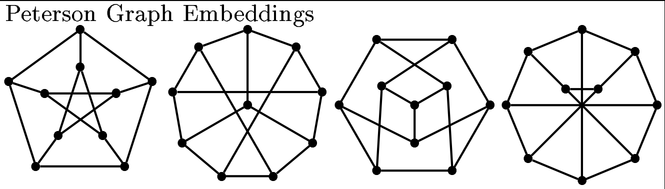
graph	$G = (V, E \subseteq V \times V)$	
complement	$\overline{G} = (V, V \times V - E)$	
Line graph	$L(G) = (E, \{\langle\langle a, b\rangle, \langle b, c\rangle\rangle\})$	
adjacency matrix	$A(G)_{ij} = [\langle i, j\rangle \in E]$	
incidence matrix	$D(G)_{ij} = [i \in j \in E(G)](-1)^{[j=\langle k,i\rangle]}$	
Laplacian	$A(L(G))[A(L(G))]^t = \Delta(V) - A$	
spectrum	$\{\lambda \mid \det(\lambda I - A(G))\}$	

contraction	$G \circ e$	multigraph with endpoints of e identified
char poly	$\chi(G; z)$	$= \det(zI - A(G))$
		$= z\chi(G - v_1; z) - \chi(G - v_1 - v_2; z), \langle v_1, v_2\rangle \in E$
chrom poly	$C(G; z)$	$= C(G - e; z) - C(G \circ e; z)$
rank poly	$R(G; x, y)$	$= \sum_{S \subseteq E} x^{r(S)} y^{s(S)}$
Tutte poly	$T(G; x, y)$	$= T(G - e; x, y) + T(G \circ e; x, y)$
	$C(G; z)$	$= z^n R(G; -u^{-1}, -1) = (-1)^{n-1} zT(G; 1 - z, 0)$

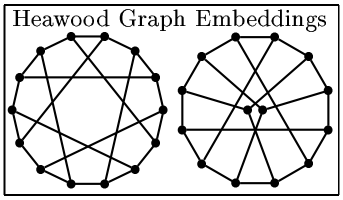
	Path P_n	Cycle C_n	Wheel W_n	Complete K_n	Bipartite $K_{n,m}$	Cube Q_n	Odd O_n
group	C_2	D_n	$D_n, n > 3$	S_n	$S_n \times S_m, n \neq m$	$C_2^n \prod S_i$	S_{2n-1}
girth	0	n	3	3	$4, (\geq 2)$	$4, (\geq 2)$	$3, 5, 6, \ldots$
diam.	n	$\lfloor \frac{n}{2} \rfloor$	2	1	2	n	$n - 1$
sp. trees	1	n	$L_{2n} - 2$	n^{n-2}	$n^{m-1} m^{n-1}$	$2^{2^n - n - 1} \prod_{k=1}^n k^{\binom{n}{k}}$	??
χ	2	$2 + [2 \mid n]$	$3 + [2 \mid n]$	n	2	2	4
$C(z)$	$z(z-1)^{n-1}$	$(z-1)^n + (-1)^n (z-1)$	$zC(C_n; z - 1)$	$z^{\underline{n-1}}$	$\sum_k \left\{ {m \atop k} \right\} (z-k)^n z^{\underline{k}}$	??	??
$\chi(z) = 0$	$\left\lvert {2\cos k\pi/(n+1) \atop 1} \right\rvert$	$\left\lvert {2 \;\; 2\cos 2k\pi/n \atop 1 \;\; 2} \right\rvert$	$\left\lvert {1 \pm \sqrt{n} \;\; 2\cos 2k\pi/n \atop 1 \;\; 2} \right\rvert$	$\left\lvert {n-1 \;\; -1 \atop 1 \;\; n-1} \right\rvert$	$\left\lvert {\sqrt{n \cdot m} \;\; -\sqrt{n \cdot m} \;\; 0 \atop 1 \;\; 1 \;\; n+m-2} \right\rvert$	$\left\lvert {n - 2k \atop \binom{n}{k}} \right\rvert$	$\left\lvert {(-1)^k (n-k) \atop \frac{2(n-k)}{2n-k}\binom{2n-1}{k}} \right\rvert$



Peterson Graph Embeddings



Heawood Graph Embeddings



Random Graph Thresholds	
comps $k \geq 3$	$n^{-3/2}$
tree $k \geq 3$	$n^{\frac{-k}{k-1}}$
cycle $k \geq 3$	n^{-1}
clique of size k	$n^{-\frac{2}{k-1}}$
connected	$\frac{\log n}{n}$
hamiltonian	$\frac{\log n(1+\epsilon)}{n}$

cover	$a \lessdot b \Leftarrow a \prec b \wedge (a < x \preceq b \rightarrow x = b)$
0,1	$\forall a \in P, 0 \preceq a \preceq 1$
interval	$[a, b] = \{x \in P \mid a \leq x \leq b\}$
lub,inf	$a \sqcup b,\ a \preceq b \equiv a \sqcup b = b$
glb,sup	$a \sqcap b,\ a \preceq b \equiv a \sqcap b = a$
irreducible	
distrib (1)	$(x \sqcap y) \sqcup (x \sqcap z) = x \sqcap (y \sqcup z)$
modular (2)	$(x \sqcap y) \sqcup (x \sqcap z) = (x \sqcap (y \sqcup (x \sqcap z)))$
semimod (3)	$a, b \succ c \rightarrow a \sqcup b \succ a, b$
atomic (4)	all elts sup of atoms (elts covering 0)
geom (5)	3,4, no inf chains
comp (6) of x	$x \sqcup x' = 1,\ x \sqcap x' = 0$

name		relation	gens	$W(k)$	$\chi(z)$	properties
Chain	$\mathcal{C}(n)$	$x < y, \in \mathbb{N}$	0,plus	1	$z^{n-1}(z-1)$	distrib
Boolean	$\mathcal{B}(n)$	$x \subseteq y$	n elts	$\binom{n}{k}$	$(z-1)^n$	1,uni6,4,co4,5
Set Partition	$\Pi(n)$	refinement	atoms	$\left\{ {n \atop k} \right\}$	$(z-1)^{\underline{n-1}}$	geometric
Divisor	$\mathcal{T}(n)$	$x \mid y \mid n$	primes	$\binom{\tau(n)}{k}$	$\prod_{p_k \mid n} \frac{1}{1 - z^{m_k}}$	modular
Dominance	$\mathcal{D}(n)$	$x_1 + y_2 < a$	n 1's	$p_{n,k}$		
Young	$\mathcal{Y}$	$x_i \leq y_i$	1,+1	P_k	$\prod_k (1 - z^k)^{-1}$	distrib
Vector Space	$\mathcal{L}(n, q)$	subspace		$\binom{n}{k}_q$	$\prod^n (z - q^k)$	2,rel6,4,co4,5

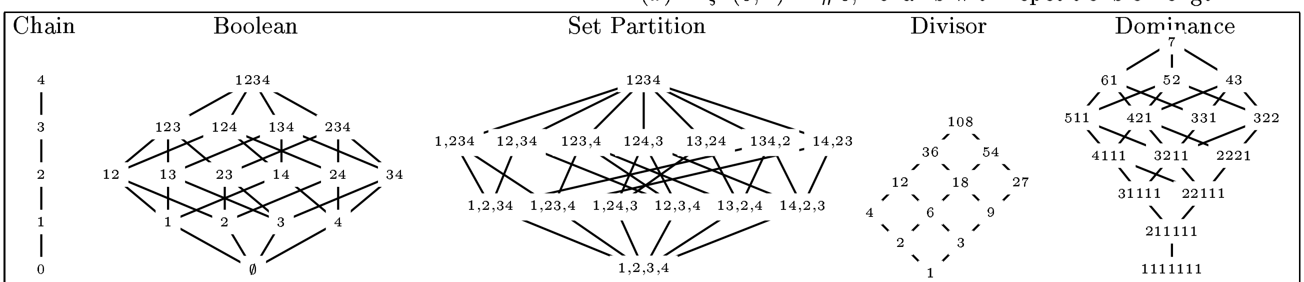
$\delta(x, y) : [x = y],\ \zeta(x, y) : [x \preceq y],\ \kappa(x, y) : [x \lessdot y]$

$\mu(x, y) : \mu\zeta = \delta, \mu = \zeta^{-1}\ \ \zeta = (\delta - \kappa)^{-1}$

$\sum_{i \geq 0} (\zeta - \delta)^i (a, b) = (2\delta - \kappa)^{-1}$ = # a,b chains

$\sum_{i \geq 0} \kappa^i (a, b) = (\delta - \kappa)^{-1}$ = # maximal a,b chains

$Z(x) = \zeta^x(0, 1)$ = #0,1 chains with repetitions of length x



F_n Fibonacci bijections

sequences of 1,2 adding to $n - 1$
subsets of $[n - 2]$ no two consecutive
compositions of $n + 1$ into parts greater than 1
reflection paths against 3 lines
compositions of n into odd parts
order ideals of the "zigzag" poset

C_n Catalan bijections

ordered binary trees with $n + 1$ leaves (n internal nodes)
ordered trees with $n + 1$ nodes
dissections of a polygon
non-crossing set partitions of n
length n sequences of ± 1 partial sums ≥ 0
l,r paths from $(0, 0)$ to (n, n), $x \geq y$
linear extensions of $2 \times n$
(2,1,3) avoiding permutations of n

$p_{n,k}$ Partition bijections

$\sum_{n \geq 0} p(n) x^n = \prod_{k \geq 1} \frac{1}{1 - x^k}$
$p(n) = \frac{1}{n} \sum_{k \geq 1} \sigma(k) p(n - k)$
distinct power of 2 parts = integers:
$\prod_{k \geq 0} (1 + x^{2^k}) = \frac{1}{1 - x}$
no parts equal to 1 = $p(n) - p(n - 1)$
largest part k with exactly k parts
odd parts = distinct parts:
$\prod_{k \geq 0} \frac{1}{1 - x^{2k+1}} = \prod_{k \geq 1} (1 + x^k)$
distinct odd parts = self conjugate
number of 1's in all partitions of n = number of distinct parts
$\prod_{k \geq 1} (1 - x^k) = \sum_k (-1)^k x^{k(3k+1)/2}$
$\prod_{k \geq 1} (1 - x^{2k})(1 + x^{2k-1} z^2)(1 - x^{2k-1} z^{-2}) = \sum_k x^{k^2} z^{2k}$
$\left(\prod_{k \geq 1} (1 - x^k)\right)^3 = \sum_{k \geq 0} (-1)^k x^{\binom{k+1}{2}} (2k + 1)$
$\sum_{k \geq 0} \frac{x^{k^2 + ak}}{\prod_{j=1}^k 1 - x^j} = \prod_{k \geq 0} \frac{1}{(1 - x^{5k+1+a})(1 - x^{5k+4-a})}$

π	$= 4\sum_{n\ge 0}\frac{(-1)^n}{2n+1}$	$= 4\sum_{n\ge 1}\tan^{-1} F_{2n+1}$	$\approx$	3.14159265358979323846264338328
e	$= \sum_{n\ge 0}\frac{1}{n!}$	$= \lim_{n\to\infty}(1+\frac{1}{n})^n$	$\approx$	2.71828182845904523536028747135
γ	$= 1+\sum_{n=2}^{\infty}\frac{1-\zeta(n)}{n}$	$= \lim_{n\to\infty}(H_n - \ln n)$	$\approx$	0.57721566490153286060651209008
ϕ	$= \frac{1+\sqrt{5}}{2}$	$= \lim_{n\to\infty} x_{n+1} = \frac{1}{1+x_n}$	$\approx$	1.61803398874989484820458683437

n		$\phi(n)$	$\sigma(n)$	$\tau(n)$	$p(n)$	n		$\phi(n)$	$\sigma(n)$	$\tau(n)$	$p(n)$	n^2	n^3	$2^{n-1}3^m$							
1		1	1	1	2	51	$3^1 17^1$	32	72	4	233	1	1	1	3	9	27	81	243	729	2187
2	2^1	1	3	2	3	52	$2^2 13^1$	24	98	6	239	4	8	2	6	18	54	162	486	1458	4374
3	3^1	2	4	2	5	53	53^1	52	54	2	241	9	27	4	12	36	108	324	972	2916	8748
4	2^2	2	7	3	7	54	$2^1 3^3$	18	120	8	251	16	64	8	24	72	216	648	1944	5832	
5	5^1	4	6	2	11	55	$5^1 11^1$	40	72	4	257	25	125	16	48	144	432	1296	3888		
6	$2^1 3^1$	2	12	4	13	56	$2^3 7^1$	24	120	8	263	36	216	32	96	288	864	2592	7776		
7	7^1	6	8	2	17	57	$3^1 19^1$	36	80	4	269	49	343	64	192	576	1728	5184			
8	2^3	4	15	4	19	58	$2^1 29^1$	28	90	4	271	64	512	128	384	1152	3456				
9	3^2	6	13	3	23	59	59^1	58	60	2	277	81	729	256	768	2304	6912				
10	$2^1 5^1$	4	18	4	29	60	$2^2 3^1 5^1$	16	168	12	281	100	1000	512	1536	4608					
11	11^1	10	12	2	31	61	61^1	60	62	2	283	121	1331	1024	3072	9216					
12	$2^2 3^1$	4	28	6	37	62	$2^1 31^1$	30	96	4	293	144	1728	2048	6144						
13	13^1	12	14	2	41	63	$3^2 7^1$	36	104	6	307	169	2197	4096	12288						
14	$2^1 7^1$	6	24	4	43	64	2^6	32	127	7	311	196	2744	8192	24576						
15	$3^1 5^1$	8	24	4	47	65	$5^1 13^1$	48	84	4	313	225	3375	16384							
16	2^4	8	31	5	53	66	$2^1 3^1 11^1$	20	144	8	317	256	4096	32768							
17	17^1	16	18	2	59	67	67^1	66	68	2	331	289	4913	65336							
18	$2^1 3^2$	6	39	6	61	68	$2^2 17^1$	32	126	6	337	324	5832								
19	19^1	18	20	2	67	69	$3^1 23^1$	44	96	4	347	361	6859								
20	$2^2 5^1$	8	42	6	71	70	$2^1 5^1 7^1$	24	144	8	349	400	8000								
21	$3^1 7^1$	12	32	4	73	71	71^1	70	72	2	353	441									
22	$2^1 11^1$	10	36	4	79	72	$2^3 3^2$	24	195	12	359	484									
23	23^1	22	24	2	83	73	73^1	72	74	2	367	529									
24	$2^3 3^1$	8	60	8	89	74	$2^1 37^1$	36	114	4	373	576									
25	5^2	20	31	3	97	75	$3^1 5^2$	40	124	6	379	625									
26	$2^1 13^1$	12	42	4	101	76	$2^2 19^1$	36	140	6	383	676									
27	3^3	18	40	4	103	77	$7^1 11^1$	60	96	4	389	729									
28	$2^2 7^1$	12	56	6	107	78	$2^1 3^1 13^1$	24	168	8	397	784									
29	29^1	28	30	2	109	79	79^1	78	80	2	401	841									
30	$2^1 3^1 5^1$	8	72	8	113	80	$2^4 5^1$	32	186	10	409	900									
31	31^1	30	32	2	127	81	3^4	54	121	5	419	961									
32	2^5	16	63	6	131	82	$2^1 41^1$	40	126	4	421	1024									
33	$3^1 11^1$	20	48	4	137	83	83^1	82	84	2	431	1089									
34	$2^1 17^1$	16	54	4	139	84	$2^2 3^1 7^1$	24	224	12	433	1156									
35	$5^1 7^1$	24	48	4	149	85	$5^1 17^1$	64	108	4	439	1225									
36	$2^2 3^2$	12	91	9	151	86	$2^1 43^1$	42	132	4	443	1296									
37	37^1	36	38	2	157	87	$3^1 29^1$	56	120	4	449	1369									
38	$2^1 19^1$	18	60	4	163	88	$2^3 11^1$	40	180	8	457	1444									
39	$3^1 13^1$	24	56	4	167	89	89^1	88	90	2	461	1521									
40	$2^3 5^1$	16	90	8	173	90	$2^1 3^2 5^1$	24	234	12	463	1600									
41	41^1	40	42	2	179	91	$7^1 13^1$	72	112	4	467	1681									
42	$2^1 3^1 7^1$	12	96	8	181	92	$2^2 23^1$	44	168	6	479	1764									
43	43^1	42	44	2	191	93	$3^1 31^1$	60	128	4	487	1849									
44	$2^2 11^1$	20	84	6	193	94	$2^1 47^1$	46	144	4	491	1936									
45	$3^2 5^1$	24	78	6	197	95	$5^1 19^1$	72	120	4	499	2025									
46	$2^1 23^1$	22	72	4	199	96	$2^5 3^1$	32	252	12	503	2116									
47	47^1	46	48	2	211	97	97^1	96	98	2	509	2209									
48	$2^4 3^1$	16	124	10	223	98	$2^1 7^2$	42	171	6	521	2304									
49	7^2	42	57	3	227	99	$3^2 11^1$	60	156	6	523	2401									
50	$2^1 5^2$	20	93	6	229	100	$2^2 5^2$	40	217	9	541	2500									

	0	1	2	3	4	5	6	7	8	9	10
Motzkin	1	1	2	4	9	21	51	127	323	835	2188
Schröder	1	2	6	22	90	394	1806	8558	41586	206098	1037718
free trees	0	1	1	1	2	3	6	11	23	47	106
rooted trees	0	1	1	2	4	9	20	48	115	286	719
graphs	0	1	2	4	11	34	156	1044	12346	274668	12005168
con. graphs	0	1	1	2	6	21	112	853	11117	261080	11716571
digraphs	0	1	3	16	218	9608	1540944	882033440	1793359192848		
con. digraphs	0	1	2	13	199	9364	1530843	880471142	1792473955306		
functional digraphs	0	0	1	2	6	13	40	100	291	797	2273
functions	0	1	3	7	19	47	130	343	951	2615	7318
posets	1	1	2	5	16	63	318	2045	16999	183231	2567284
topologies	1	1	3	9	33	139	718	4535			
tournaments	1	1	2	4	12	56	456	6880	191536	9733056	903753248

n	2^n	F_n	C_n	B_n	H_n	P_n
0	1	0	1	1	0	1
1	2	1	1	1	1	1
2	4	1	2	2	$\frac{3}{2}$	2
3	8	2	5	5	$\frac{11}{6}$	3
4	16	3	14	15	$\frac{25}{12}$	5
5	32	5	42	52	$\frac{137}{60}$	7
6	64	8	132	203	$\frac{49}{20}$	11
7	128	13	429	877	$\frac{363}{140}$	15
8	256	21	1430	4140	$\frac{761}{280}$	22
9	512	34	4862	21147	$\frac{7129}{2520}$	30
10	1024	55	16796	115975	$\frac{7381}{2520}$	42
11	2048	89	58786	678570	$\frac{83711}{27720}$	56
12	4096	144	208012	4213597	$\frac{86021}{27720}$	77
13	8192	233	742900	27644437	$\frac{1145993}{360360}$	101
14	16384	377	2674440	190899322	$\frac{1171733}{360360}$	135
15	32768	610	9694845	1382958545	$\frac{1195757}{360360}$	176
16	65536	987	35357670	10480142147	$\frac{2436559}{720720}$	231
17	131072	1597	129644790	82864869804	$\frac{42142223}{12252240}$	297
18	262144	2584	477638700	682076806159	$\frac{14274301}{4084080}$	385
19	524288	4181	1767263190	5832742205057	$\frac{275295799}{77597520}$	490
20	1048576	6765	6564120420	51724158235372	$\frac{55835135}{15519504}$	627

$\binom{n}{m}$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

$\left[{n \atop m}\right]$	1	2	3	4	5	6	7	8	9	10
1	1									
2	1	1								
3	2	3	1							
4	6	11	6	1						
5	24	50	35	10	1					
6	120	274	225	85	15	1				
7	720	1764	1624	735	175	21	1			
8	5040	13068	13132	6769	1960	322	28	1		
9	40320	109584	118124	67284	22449	4536	546	36	1	
10	362880	1026576	1172700	723680	269325	63273	9450	870	45	1

$\left\{{n \atop m}\right\}$	1	2	3	4	5	6	7	8	9	10
1	1									
2	1	1								
3	1	3	1							
4	1	7	6	1						
5	1	15	25	10	1					
6	1	31	90	65	15	1				
7	1	63	301	350	140	21	1			
8	1	127	966	1701	1050	266	28	1		
9	1	255	3025	7770	6951	2646	462	36	1	
10	1	511	9330	34105	42525	22827	5880	750	45	1

$\left\langle{n \atop m}\right\rangle$	0	1	2	3	4	5	6	7	8
0	1								
1	1								
2	1	1							
3	1	4	1						
4	1	11	11	1					
5	1	26	66	26	1				
6	1	57	302	302	57	1			
7	1	120	1191	2416	1191	120	1		
8	1	247	4293	15619	15619	4293	247	1	
9	1	502	14608	88234	156190	88234	14608	502	1

$d_{n,m}$	0	1	2	3	4	5	6	7	8	9
0	1									
1	0	1								
2	1	0	1							
3	2	3	0	1						
4	9	8	6	0	1					
5	44	45	20	10	0	1				
6	265	264	135	40	15	0	1			
7	1854	1855	924	315	70	21	0	1		
8	14833	14832	7420	2464	630	112	28	0	1	
9	133496	133497	66744	22260	5544	1134	168	36	0	1

$p_{n,m}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1																					
2	1	1																				
3	1	1	1																			
4	1	2	1	1																		
5	1	2	2	1	1																	
6	1	3	3	2	1	1																
7	1	3	4	3	2	1	1															
8	1	4	5	5	3	2	1	1														
9	1	4	7	6	5	3	2	1	1													
10	1	5	8	9	7	5	3	2	1	1												
11	1	5	10	11	10	7	5	3	2	1	1											
12	1	6	12	15	13	11	7	5	3	2	1	1										
13	1	6	14	18	18	14	11	7	5	3	2	1	1									
14	1	7	16	23	23	20	15	11	7	5	3	2	1	1								
15	1	7	19	27	30	26	21	15	11	7	5	3	2	1	1							
16	1	8	21	34	37	35	28	22	15	11	7	5	3	2	1	1						
17	1	8	24	39	47	44	38	29	22	15	11	7	5	3	2	1	1					
18	1	9	27	47	57	58	49	40	30	22	15	11	7	5	3	2	1	1				
19	1	9	30	54	70	71	65	52	41	30	22	15	11	7	5	3	2	1	1			
20	1	10	33	64	84	90	82	70	54	42	30	22	15	11	7	5	3	2	1	1		
21	1	10	37	72	101	110	105	89	73	55	42	30	22	15	11	7	5	3	2	1	1	
22	1	11	40	84	119	136	131	116	94	75	56	42	30	22	15	11	7	5	3	2	1	1

[1] M. Abramowitz and I. A. Stegun, editors. *Handbook of mathematical functions with formulas, graphs, and mathematical tables.* Dover Publications Inc., New York, 1992. Reprint of the 1972 edition.

[2] M. Aigner. *Combinatorial Theory.* Springer-Verlag, Berlin, 1979.

[3] N. Biggs. *Algebraic Graph Theory.* Cambridge University Press, 2nd edition, 1993.

[4] L. Comtet. *Advanced Combinatorics.* Reidel, Dordrecht, 1974.

[5] I. P. Goulden and D. M. Jackson. *Combinatorial Enumeration.* John Wiley and Sons, Inc., New York, 1983.

[6] R. L. Graham, D. E. Knuth, and O. Patashnik. *Concrete Mathematics.* Addison-Wesley, Reading, MA, 1989.

[7] G. H. Hardy and E. M. Wright. *An Introduction to the Theory of Numbers.* Oxford, 5th edition, 1979.

[8] D. E. Knuth. *The Art of Computer Science: Fundamental Algorithms*, volume I. Addison-Wesley, Reading, MA, 1973.

[9] J. Riordan. *An Introduction to Combinatorial Analysis.* John Wiley and Sons, Inc., New York, 1958.

[10] J. Riordan. *Combinatorial Identities.* John Wiley and Sons, Inc., New York, 1968.

[11] N. J. A. Sloane and S. Plouffe. *The Encyclopedia of Integer Sequences.* Academic Press Inc., San Diego, CA, 1995.

[12] R. P. Stanley. *Enumerative combinatorics. Vol. I.* Wadsworth & Brooks/Cole Advanced Books & Software, Monterey, Calif., 1986. With a foreword by Gian-Carlo Rota.

[13] H. S. Wilf. *generatingfunctionology.* Academic Press, Inc., Boston, 1994.