The Sum-Capacity of the Ergodic Fading Gaussian Cognitive Interference Channel

Diana Maamari, Natasha Devroye, and Daniela Tuninetti

***Abstract*—This paper characterizes the sum-capacity of the ergodic fading Gaussian overlay cognitive interference channel (EGCIFC), a time-varying channel with two source–destination pairs in which a primary/licensed transmitter and a secondary/cognitive transmitter share the same spectrum and where the cognitive transmitter has noncausal knowledge of the primary user's message. The throughput/sum-capacity is characterized under the assumption of perfect knowledge of the instantaneous fading states at all terminals, which are assumed to form an ergodic process. A genie-aided outer bound on the sum-capacity is developed and then matched with an achievable scheme, thereby completely characterizing the sum-capacity of the EGCIFC. The power allocation policy that maximizes the sum-capacity is derived. It is shown that the sum-capacity achieving scheme for an EGCIFC is "separable" in all regimes (i.e., coding across fading states is not necessary), as opposed to the classical interference channel. Extensions to the whole capacity region are discussed. As a capacity achieving scheme for the EGCIFC under certain channel gain conditions, and as a topic of independent interest, the ergodic capacity of a point-to-point multiple-input–single-output channel with per-antenna power constraints and with perfect channel state information at all terminals is also derived.**

***Index Terms*—Cognitive radio, ergodic capacity, power allocation, separability, convex optimization.**

I. Introduction

There are two important aspects that make wireless communication challenging. The first is *fading*, that is, the time variation of the channel gains due to small scale effects, such as those due to multi-path, and large-scale effects, such as those due to path loss and shadowing. The second is *interference* between wireless users communicating over the same frequency band. In this work we focus on the two-user fading interference channel where one of the transmitters is cognitive, or knows the message of the other independent transmitter. This channel model thus experiences both fading and interference, as well as a third phenomena seen in wireless communications: the ability of transmit nodes to *cooperate*. We aim to characterize the sum-capacity/throughput of this channel so as to highlight the impact of fading, interference and asymmetric cooperation between transmitters.

Cognitive networks are wireless networks in which certain nodes are cognitive radios, or artificially intelligent devices, and have been the subject of intensive investigation by the wireless communication community in the past decade. In a cognitive network, cognitive/secondary transmitters share the spectrum with primary/licensed users. The cognitive devices exploit *side information* about their environment to improve spectral management. Depending on the nature of the side information [2], cognitive users either search for unused spectrum (interweave), or operate simultaneously with non-cognitive transmitters as long as the interference produced is within an acceptable threshold (underlay), or relay part of the primary user's message and cancel interference through advanced encoding schemes (overlay). In this paper we consider the overlay paradigm where a primary transmitter–receiver (Tx–Rx) pair share the same spectrum with a cognitive Tx–Rx pair. According to the overlay paradigm [2], the secondary Tx is assumed to have non-causal knowledge of the message of the primary Tx.

The channel between the Txs and the Rxs is assumed to experience ergodic fading (i.e., time average of every sufficiently long fading realization equals the statistical average). All nodes in the network are assumed to have perfect instantaneous knowledge of the channel fading coefficients, or full channel state information (CSI). Although the full CSI assumption might be impractical even in the presence of a dedicated feedback channel from the Rxs to the Txs, the resulting model serves as the customary first step towards understanding the performance of more realistic models. Under these assumptions we seek to determine the sum-capacity of the network and the corresponding optimal power allocation policy under a long-term average transmit power constraint at the Txs. In doing so, we also seek to answer the question of whether coding separately across fading state is optimal [3].

This channel model, which we term the *ergodic fading Gaussian overlay cognitive interference channel* (EGCIFC), is practically motivated as follows:

- Channels with Retransmission. Consider a network of primary and cognitive users in which a fixed-rate, i.e., not a function of the channel gains, primary user's message was sent but was not decoded at the intended receiver. The primary receiver informs the primary transmitter by sending a NACK, and a retransmission takes place. If the cognitive transmitter overheard the primary's initial

Manuscript received March 4, 2014; revised June 19, 2014 and September 3, 2014; accepted September 12, 2014. Date of publication September 25, 2014; date of current version February 6, 2015. This work was supported in part by the National Science Foundation under Award 1017436. The contents of this paper are solely the responsibility of the authors and do not necessarily represent the official views of the NSF. The results in this paper were presented in part at the IEEE International Symposium on Information Theory, Honolulu, HI, USA, June 2014. The associate editor coordinating the review of this paper and approving it for publication was L. K. Rasmussen.

The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL 60607 USA (e-mail: dmaama2@uic.edu; devroye@uic.edu; danielat@uic.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TWC.2014.2360382

1536-1276 © 2014 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

transmission and was able to successfully decode the message in the first round then, in the retransmission phase, the cognitive transmitter would have non-causal primary message knowledge and could transmit together with the primary in the ARQ round(s) [4].

- Coordinated Multipoint Transmission. Consider a network in which multiple transmitters (may be thought of as base-stations) are connected by high capacity backhaul links, allowing them to exchange messages to be transmitted to the receivers (may be thought of as mobile users). This model includes the EGCIFC as a special case. In general, studying networks where nodes may exchange messages through high-capacity backhaul links may reveal what type of message knowledge structure maximize the network performance [5], [6].
- Overlay Networks. This model is inspired by the idea of layered cognitive networks where the first layer consists of primary users and each additional layer consists of cognitive users that share the same spectrum. Each additional layer is given the codebook(s) of all previous layers. This hierarchical codebook knowledge enables them to causally learn the lower layers' messages and aid in their transmission. Thus studying the non-causal message knowledge setting provides an upper bound to the more realistic case where the messages are causally learned by the cognitive transmitter(s) [7].

A. Prior Work

In [8] the two-user Gaussian interference channel with ergodic fading was introduced and the optimal power allocation policy that maximizes the outer bound was investigated. In [3] the authors also considered the same model and showed that in general joint encoding and decoding across fading states is necessary to achieve capacity when perfect CSI at all nodes is assumed (note that fading is an example of the more general result that parallel interference channels are not separable [9]). However, [3] showed that in some parameter regimes, such as very strong or very weak interference, separate encoding and decoding across fading states is optimal. In other words, in interference channels separability may hold for certain channel states but not for all channel states. While interference channels are not separable in general, it is known that multi-access [10] and broadcast [11] channels with the same CSI assumption are separable. Since the EGCIFC has elements of *both* an interference channel and a broadcast channel, it is not *a priori* obvious that a separable scheme is capacity achieving. A formal proof of the optimality of separability for the EGCIFC is given in this paper.

In [12] the authors also consider a fading cognitive network under the underlay paradigm. The primary user in this case is completely oblivious to the existence of the cognitive user. The relationship between the achievable capacity of the secondary channel and the interference caused at the primary receiver was quantified. Instantaneous and average interference power constraints were both considered and the optimal power allocation policy for the secondary user in each case was derived. In this work we consider the overlay paradigm with long-term average power constraints at the primary and secondary transmitters. Moreover our primary user is not oblivious to the existence of the secondary user.

In [13] the authors consider a cognitive radio network under the underlay paradigm where primary and secondary users are subject to block fading. The primary user is not capable of adapting its power allocation while the secondary user is able to do so. The authors derive the optimal power allocation strategies for the cognitive user to maximize its ergodic and outage capacity. In our work *both* users are capable of adapting their transmit power over the different fading states based on the channel state information, and we consider the overlay paradigm.

In [14] the authors consider the fading cognitive single input single output (SISO) MAC channel for the underlay paradigm, where the secondary users are subject to both a transmit power constraint and interference power constraint to primary users. It is shown that the sum-capacity achieving power allocation policy is a water filling type of solution. The key difference between the EGCIFC and the model in [14] is the message structure; in [14] each sender only knows its own message and thus the transmit signals are independent; in the EGCIFC the transmit signals are correlated due to the non-causal primary message knowledge at the secondary transmitter. Because of this, the sum-capacity achieving power allocation policy for the EGCIFC will not be a water filling type.

As a byproduct of our analysis and as a result of independent interest, we shall show that the optimal power policy for the EGCIFC in some regimes requires both transmitters to beamform to the primary receiver; in this case the model reduces to a point-to-point multiple input single output (MISO) channel with per-antenna power constraints (PerPC). In [15] the author finds the capacity for the point-to-point MISO channel with PerPC with two different assumptions of CSI. The capacity for a constant channel with CSI at both the transmitter and receiver and that of a Rayleigh fading channel with CSI at the receiver only were derived. The optimal signaling scheme was found for both cases. The author compares the result with the capacity of a MISO channel with sum power constraints (SumPC) through numerical examples. In both cases the capacity with PerPC is, as expected, less than that with SumPC. Here we characterize the sum-capacity of the EGCIFC with CSI at the transmitters and receivers and find the optimal signaling scheme, which is thus different from the solutions found in [15].

In [16] the authors first introduced the information theoretic study of the cognitive radio channel (same as the cognitive interference channel) which falls into the overlay paradigm. In that work, the channel gains were constant and achievable rate regions and outer bounds were derived. In [17] the authors found the capacity of the cognitive interference channel in the weak interference regime. In particular, the power split which ensures that the primary receiver rate continues to be the same as that without interference from the cognitive user. For the state-of-the-art on the two-user cognitive interference channel with constant channel gains (known to all nodes) we refer the reader to [18] and [19]. In this paper we remove the assumption of constant channel gains and consider the fading (time varying) cognitive interference channel.

B. Contributions and Paper Outline

Our main contributions and paper organization are as follows. Section II presents the channel model for the EGCIFC. Section III contains the following main results:

1) We first present a sum-capacity genie-aided outer bound for the EGCIFC. This genie-aided outer bound consists of giving side-information to the cognitive receiver of primary user's message and output to the cognitive receiver, as for the non-fading case [18], [19]. We then provide a matching achievability scheme as an extension to the fading case of the scheme in [17], thus completely characterizing the sum-capacity under the assumption of perfect CSI at all nodes.
2) When the primary receiver experiences strong interference, the sum-capacity achieving scheme is that of point-to-point MISO channel with PerPC and perfect CSI at the transmitter and receiver. In [1] we have characterized the capacity of a MISO with full CSI at all nodes and with arbitrary number of antennas. In this work, we further consider the case of weak interference which was left open in [1].
3) The power allocation policy that maximizes the sum-capacity for the EGCIFC is derived and shown to depend on the relative channel gains for a given fading state. Extensions to the whole capacity region are discussed.

In Section IV we illustrate our results with numerical examples. We consider a Rayleigh fading point-to-point MISO channel with PerPC. The capacity for this channel is compared to that of a MISO channel with SumPC and that of a MISO channel with constant power allocation. We then consider a Rayleigh EGCIFC where the mean of the channel gains are chosen such that with high probability the channel is either in strong interference or in weak interference. The sum-capacity of the EGCIFC under the optimal power allocation is compared to that with constant power allocation and that of a MISO channel with PerPC (i.e., allocate power as if the channel is in strong interference even if this is not the case). We then consider a scheme where we optimize the correlation coefficient (or power split) among the inputs but keep the power allocation constant at each fading state and show that, although in principle this is a suboptimal scheme, it is almost capacity achieving for the EGCIFC—thereby highlighting the importance of correctly balancing the amount of power the secondary user uses for transmission of its own message and that for relaying the primary message.

Some proofs may be found in Appendix.

C. Notation

Throughout the paper we use the following notation:

- x^n denotes a vector of length n with components $(x_1, \ldots, x_n)$.
- $A^\dagger$ represents the transpose and complex-conjugate of the matrix A.
- $A^\star$ represents the optimal solution for a given optimization problem.
- A^N represents a vector of random variables A_i with $i \in [1:N]$.

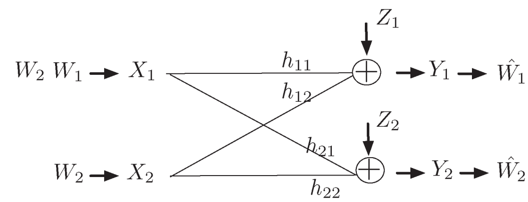

Fig. 1. Ergodic fading Gaussian cognitive interference channel (EGCIFC).

- $\mathbb{E}[.]$ denotes the expectation operator.
- $\log(.)$ denotes logarithm in base 2, $[x]^+ := \max(x, 0)$ and $\log^+(.) := \max(\log(.), 0)$.
- $I(.;.)$ denotes the mutual information and $h(.)$ denotes the differential entropy.
- $\mathbb{P}(A)$ represents the probability of event A.
- $\mathcal{N}(\mu, \sigma^2)$ denotes a complex-valued circular symmetric Gaussian random variable with mean μ and variance σ^2.

II. Channel Model

The cognitive interference channel consists of two transmit-receive pairs Tx_1 to Rx_1 and Tx_2 to Rx_2 representing the cognitive and primary users, respectively, as shown in Fig. 1. Each transmitter Tx_k wishes to convey to its destination Rx_k an independent message W_k, which is uniformly distributed over the set $[1 : 2^{NR_k}]$, where R_k is the rate in bits per channel use, and N represents the codeword length, for $k \in [1 : 2]$. Tx_1 is *cognitive* in the sense that it has non-causal message knowledge of the primary user's (Tx_2) message W_2. A rate vector (R_1, R_2) is said to be achievable if there exists a family of codes indexed by N such that the probability of decoding error can be made arbitrarily small [20]. The sum-capacity is defined as the maximum achievable $R_1 + R_2$.

In Gaussian noise and with ergodic fading, the ECIFC input-output relationship at every time instant (time index is omitted for easier notation) is given by

$$Y_1 = h_{11}X_1 + h_{12}X_2 + Z_1, \quad Z_1 \sim \mathcal{N}(0,1), \tag{1}$$

$$Y_2 = h_{22}X_2 + h_{21}X_1 + Z_2, \quad Z_2 \sim \mathcal{N}(0,1), \tag{2}$$

where $\mathbf{H} := \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ denotes the random channel gain matrix (with complex entries generated randomly at each time instant/channel use according to a known, stationary and ergodic random process), with $[\mathbf{H}]_{i,j} = h_{ij} \in \mathbb{C},\ i, j \in [1 : 2]$ representing the fading channel gain between Tx_j and Rx_i. A realization of $\mathbf{H}$ is indicated as $\mathbf{h}$. Tx_j, with channel input X_j is subject to the long-term average power constraint $\mathbb{E}[|X_j|^2] \leq \overline{P}_j,\ j \in [1 : 2]$. With CSI at all terminals, the transmitters can perform dynamic power allocation, and transmit with power $P_j(\mathbf{h}) \geq 0,\ j \in [1 : 2]$, at a channel use with fading state $\mathbf{h}$. We seek to determine the sum-capacity optimal power allocation for each user such that $\mathbb{E}[P_j(\mathbf{h})] \leq \overline{P}_j,\ j \in [1 : 2]$. Thanks to cognition, the channel inputs can be correlated; the input covariance matrix in fading state $\mathbf{h}$ is denoted by

$$\Sigma(\mathbf{h}) := \begin{bmatrix} P_1(\mathbf{h}) & \rho^\dagger(\mathbf{h})\sqrt{P_1(\mathbf{h})P_2(\mathbf{h})} \\ \rho(\mathbf{h})\sqrt{P_1(\mathbf{h})P_2(\mathbf{h})} & P_2(\mathbf{h}) \end{bmatrix}, \tag{3}$$

where the correlation coefficient must satisfy $|\rho(\mathbf{h})| \leq 1$.

III. MAIN RESULTS

This section includes the main results for the EGCIFC. A sum-capacity outer bound is presented as a maximization problem over three constraints: the two long-term average power constraints at the transmitters and the constraint on the correlation coefficient. We then prove that the outer bound with the optimal power allocation policy is achievable through a variation of the achievability scheme of [17] proposed for the constant channel gain cognitive interference channel in weak interference. The achievability scheme, unlike that for certain parameter regimes of the interference channel, is separable, that is, encoding need not be done across fading states. For a certain parameter regime (relative channel gains), the sum-capacity achieving power allocation corresponds to the optimal power allocation scheme for a point-to-point MISO channel with a PerPC. Thus as a topic of independent interest we characterize the power allocation policy and the capacity of the fading point-to-point MISO channel with PerPC and an arbitrary number of antennas.

The sum-capacity of the EGCIFC is presented next and is shown to be the solution of a maximization problem in the variables $(P_1(\mathbf{h}), P_2(\mathbf{h}), \rho(\mathbf{h}))$ in (3). We have:

Theorem 1: The ergodic sum-capacity of the EGCIFC is

$$C_{\mathrm{EGCIFC,\,sum}} = \max \mathbb{E}\Bigg[\log\left(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^{\dagger}\right) + \log\left(\frac{1+(1-|\rho(\mathbf{h})|^2)\max\{|h_{11}|^2,|h_{21}|^2\}P_1(\mathbf{h})}{1+(1-|\rho(\mathbf{h})|^2)|h_{21}|^2P_1(\mathbf{h})}\right)\Bigg], \quad (4)$$

with $\mathbf{h}_2 := [h_{21}\ h_{22}]$ (the index 2 refers to the second row of the instantaneous fading realization channel matrix $\mathbf{h}$) and where the maximization is over $P_i(\mathbf{h}) \geq 0 : \mathbb{E}[P_i(\mathbf{h})] \leq \overline{P}_i, i \in [1:2]$, and $|\rho(\mathbf{h})| \leq 1$.

Proof: As a generalization of the outer bound technique of [18], [19] to the fading case, the sum-capacity is upper-bounded by

$$\begin{aligned}
N(R_1+R_2-2\epsilon_N) &\overset{(a)}{\leq} I\left(W_1;Y_1^N|\mathbf{h}^N\right)+I\left(W_2;Y_2^N|\mathbf{h}^N\right)\\
&\overset{(b)}{\leq} I\left(W_1;Y_1^N,Y_2^N|\mathbf{h}^N,W_2\right)+I\left(W_2;Y_2^N|\mathbf{h}^N\right)\\
&\overset{(c)}{=} I\left(W_1,Y_1^N|\mathbf{h}^N,W_2,Y_2^N\right)+I\left(W_1;Y_2^N|W_2,\mathbf{h}^N\right)+I\left(W_2;Y_2^N|\mathbf{h}^N\right)\\
&\overset{(d)}{=} I\left(W_1;Y_1^N|\mathbf{h}^N,W_2,Y_2^N\right)+I\left(W_1,W_2;Y_2^N|\mathbf{h}^N\right)\\
&\overset{(e)}{\leq} I\left(X_1^N;Y_1^N|\mathbf{h}^N,X_2^N,Y_2^N\right)+I\left(X_1^N,X_2^N;Y_2^N|\mathbf{h}^N\right)\\
&\overset{(f)}{=} h\left(Y_1^N|\mathbf{h}^N,X_2^N,Y_2^N\right)-h\left(Y_1^N|\mathbf{h}^N,X_2^N,Y_2^N,X_1^N\right)+h\left(Y_2^N|\mathbf{h}^N\right)-h\left(Y_2^N|\mathbf{h}^N,X_1^N,X_2^N\right)\\
&\overset{(g)}{=} \sum_{i=1}^N h\left(Y_{1i}|\mathbf{h}^N,X_2^N,Y_2^N,(Y_1)_1^{i-1}\right) - h\left(Y_{1i}|\mathbf{h}^N,X_2^N,Y_2^N,X_1^N,(Y_1)_1^{i-1}\right)+h\left(Y_{2i}|\mathbf{h}^N,(Y_2)_1^{i-1}\right) - h\left(Y_{2i}|\mathbf{h}^N,X_1^N,X_2^N,(Y_2)_1^{i-1}\right)\\
&\overset{(h)}{\leq} \sum_{i=1}^N h(Y_{1i}|\mathbf{h}_i,X_{2i},Y_{2i})-h\left(Y_{1i}|\mathbf{h}_i,X_{2i},Y_{2i},X_{1i}\right)+h(Y_{2i}|\mathbf{h}_i)-h(Y_{2i}|\mathbf{h}_i,X_{1i},X_{2i})\\
&\overset{(i)}{=} \sum_{i=1}^N I(X_{1i};Y_{1i}|X_{2i},Y_{2i},\mathbf{h}_i)+I(X_{1i},X_{2i};Y_{2i}|\mathbf{h}_i)\\
&\overset{(j)}{\leq} \sum_{i=1}^N I(X_{1Gi};Y_{1i}|Y_{2i},X_{2Gi},\mathbf{h}_i)+I(X_{1Gi},X_{2Gi};Y_{2i}|\mathbf{h}_i),
\end{aligned}$$

where the different inequalities follow from: (a) Fano's inequality (here $\epsilon_N \to 0$ as $N \to \infty$), (b) a genie provides side information (Y_2^N, W_2) to Rx_1 and independence of messages, (c) chain rule for mutual information, (d) recombining mutual information terms, (e) data processing inequality and definition of encoding functions, (f) definition of mutual information, (g) chain rule for entropy, (h) conditioning reduces entropy and memoryless channel, (i) definition of mutual information, and (j) Gaussian maximizes entropy, where X_{1Gi}, X_{2Gi} are jointly Gaussian with the same covariance matrix as X_{1i}, X_{2i}. The dependence on the time index i can be eliminated by taking the appropriate limit over N as done in [3]. We note that in (g) we can choose the correlation coefficient among Z_1 and Z_2 since the Rxs do not cooperate and hence the capacity region only depends on the noise marginal distributions (i.e., we can choose the worst noise correlation as long as the marginal distributions are preserved). From the results on the static channel [19] we know that the worst noise correlation is $\min\left\{\frac{|h_{11}|}{|h_{21}|},\frac{|h_{21}|}{|h_{11}|}\right\}$. With this worst noise correlation and with the input covariance as in (3), the sum-capacity outer bound in (g) for the EGCIFC can be expressed as in (4).

We now demonstrate that the derived outer bound is achievable. A variable rate coding scheme is used: in this case at each channel use (each coordinate of the codewords X_1^N and X_2^N), the optimal powers for that particular fading state are used $P_1^\star(\mathbf{h})$ and $P_2^\star(\mathbf{h})$ by the secondary and primary nodes, respectively with an optimal $\rho^\star(\mathbf{h})$. The cognitive transmitter assigns part of its power to relay W_2 and uses the remaining power to send its own message by Dirty Paper Coding (DPC) [21] against W_2, which it knows non-causally. Similar to the scheme for the static channel [17] with the difference that at each channel use, the optimal parameters $P_1^\star(\mathbf{h})$, $P_2^\star(\mathbf{h})$ and $\rho^\star(\mathbf{h})$ for that particular fading state that maximize (4) are used. In particular, let U_1 and U_2 be independent Gaussian random variables with zero mean and unit variance, and

- Primary user sends

$$X_2 = \sqrt{P_2^\star(\mathbf{h})}U_2, \quad (5a)$$

- Cognitive user sends $X_1 = X_{2\mathrm{R}} + X_{1\mathrm{DPC}}$ where

$$X_{2\mathrm{R}} = \sqrt{|\rho^\star(\mathbf{h})|^2 P_1^\star(\mathbf{h})}e^{\mathrm{j}(-\angle h_{21}+\angle h_{22})}U_2, \quad (5b)$$

$$X_{1\mathrm{DPC}} = \sqrt{\left(1-|\rho^\star(\mathbf{h})|^2\right)P_1^\star(\mathbf{h})}U_1, \quad (5c)$$

and where $X_{1\mathrm{DPC}}$ is DPC against the 'non-causally known state'

$$S = \left(h_{12}\sqrt{P_2^\star(\mathbf{h})}+h_{11}\sqrt{|\rho^\star(\mathbf{h})|^2P_1^\star(\mathbf{h})}e^{\mathrm{j}(-\angle h_{21}+\angle h_{22})}\right)U_2.$$

- The received signals are

$$Y_1 = h_{11}\sqrt{\left(1-|\rho^\star(\mathbf{h})|^2\right)P_1^\star(\mathbf{h})}U_1 + S + Z_1,$$

$$Y_2 = e^{\mathrm{j}\angle h_{22}}\left(|h_{22}|\sqrt{P_2^\star(\mathbf{h})} + |h_{21}|\sqrt{|\rho^\star(\mathbf{h})|^2 P_1^\star(\mathbf{h})}\right)U_2 + h_{21}\sqrt{\left(1-|\rho^\star(\mathbf{h})|^2\right)P_1^\star(\mathbf{h})}U_1 + Z_2; \quad (5d)$$

- Since Tx_1 used DPC we have

$$R_1 = \log\left(1+|h_{11}|^2\left(1-|\rho^\star(\mathbf{h})|^2\right)P_1^\star(\mathbf{h})\right), \quad (5e)$$

and if Rx_2 treats U_1 as noise we have

$$R_2 = \log\left(1+\frac{\left(|h_{21}|\,|\rho^\star(\mathbf{h})|\,\sqrt{P_1^\star(\mathbf{h})} + |h_{22}|\sqrt{P_2^\star(\mathbf{h})}\right)^2}{1+|h_{21}|^2\left(1-|\rho^\star(\mathbf{h})|^2\right)P_1^\star(\mathbf{h})}\right). \quad (5f)$$

By summing (5e) and (5f), re-arranging and taking the expectation yields the sum-rate in (4) when $|h_{11}|^2 \geq |h_{21}|^2$. When $|h_{11}|^2 < |h_{21}|^2$ the sum-rate in (4) in maximized by $|\rho^\star(\mathbf{h})| = 1$ and is again achievable by (5e) and (5f). ■

Remark 1: One can think of parallel Gaussian cognitive interference channels (PGCIFC) as an EGCIFC in which each sub-channel occurs with equal probability and so the sum-capacity result described in this paper gives also the sum-capacity for PGCIFC.

While Theorem 1 expresses the ergodic sum-capacity as an optimization problem, we now proceed to determine the optimal power allocation policy. To do so, we first investigate a topic of independent interest: the ergodic capacity of the point-to-point MISO channel with PerPC, which gives the sum-capacity of the EGCIFC when $|h_{11}|^2 < |h_{21}|^2$.

A. The Ergodic Capacity of the Point-to-Point MISO Channel With Per-Antenna Power Constraints and Perfect CSI at All Terminals

The MISO channel with n transmit antennas with PerPC has output

$$Y = [H_1\ H_2 \cdots H_n]\,\mathbf{X} + Z \in \mathbb{C}, \quad Z \sim \mathcal{N}(0,1),$$

where each entry of the input vector $\mathbf{X} := [X_1, \cdots, X_n]^T$ has a separate long-term average transmit power constraint $\mathbb{E}[|X_i|^2] \leq \overline{P}_i$, for $i \in [1:n]$. The channel vector $[H_1\ H_2 \cdots H_n]$ has complex-valued entries representing the channel gain coefficient from each transmit antenna to the receive antenna and is generated from an ergodic process whose instantaneous realization is known to the transmitter and the receiver. We aim to characterize the ergodic capacity of this channel, where capacity is defined as usual [20]. In the following, we denote the instantaneous realization of the channel vector as $\mathbf{h} := [h_1 \cdots\ h_n] \in \mathbb{C}^n$ and the power allocated on antenna i in fading realization $\mathbf{h}$ as $P_i(\mathbf{h}), i \in [1:n]$.

Theorem 2: The ergodic capacity of the Gaussian fading MISO channel with PerPC is

$$C_{\mathrm{MISOPerPC}} = \max\ \mathbb{E}\left[\log\left(1+\left(\sum_{i\in[1:n]}|h_i|\sqrt{P_i(\mathbf{h})}\right)^2\right)\right] \quad (6)$$

where the maximization in (6) is over $P_i(\mathbf{h}) \geq 0 : \mathbb{E}[P_i(\mathbf{h})] \leq \overline{P}_i,\ i \in [1:n]$. The optimal power allocation policy is given by

$$P_j^\star(\mathbf{h}) = \frac{\left[\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i} - 1\right]^+}{\left(\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}\right)^2}\,\frac{|h_j|^2}{\lambda_j^2}, \quad (7)$$

where the Lagrange multipliers $\{\lambda_i,\ i \in [1:n]\}$ solve the non-linear system of equations $\mathbb{E}[P_j^\star(\mathbf{h})] = \overline{P}_j,\ j \in [1:n]$, and attains

$$C_{\mathrm{MISOPerPC}} = \mathbb{E}\left[\log^+\left(\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}\right)\right]. \quad (8)$$

Proof: The proof is based on solving the dual problem to (6) and is provided in Appendix A. The capacity in (8) can be obtained by *beamforming*: each antenna transmits

$$X_i = \exp\{-\mathrm{j}\angle h_i\}\sqrt{P_i^\star(\mathbf{h})}\,U,\ U \sim \mathcal{N}(0,1),\ i \in [1:n]$$

where $P_i^\star(\mathbf{h})$ is the optimal power allocation given by (7). By taking the average over all fading states, the capacity can be expressed as (8). ■

Remark 2: If Lagrange multipliers in (7) are all equal to λ, then the power allocation becomes

$$P_i^\star(\mathbf{h}) = \left[\frac{1}{\lambda} - \frac{1}{\|\mathbf{h}\|^2}\right]^+\frac{|h_i|^2}{\|\mathbf{h}\|^2}, \quad (9)$$

with $\|\mathbf{h}\|^2 =: \sum_{\in[1:n]}|h_i|^2$. The expression in (9) corresponds to the water-filling power allocation optimal under SumPC, in which case the Lagrange multiplier would satisfy $\mathbb{E}\left[\sum_{i\in[1:n]}P_i^\star(\mathbf{h})\right] = \mathbb{E}\left[\left[\frac{1}{\lambda} - \frac{1}{\|\mathbf{h}\|^2}\right]^+\right] = \sum_{i\in[1:n]}\overline{P}_j$. This can happen if the power constraint on each antenna is the same and the distribution of the fading vector does not change by permuting its components, such as with identical and independent distributed fading.

Remark 3: In [15] the capacity of the fading MISO channel with PerPC was derived analytically under the assumption of CSI at the receiver only—in Theorem 2 we consider the case of CSI at both the transmitter and receiver, and obtain the capacity with PerPC in closed-form.

Determining analytically the capacity under PerPC is elusive because the corresponding Lagrangian dual problem does not seem to lead to a closed form solution in general. For example, in [22] the authors considered a MIMO-MAC with per-antenna power constraints and could only find efficient algorithms to solve numerically the problem of finding the optimal input covariance matrices that maximize the sum-capacity. The capacity of the static point-to-point MIMO channel with PerPC

TABLE I
OPTIMAL POWER POLICY WHEN $|h_{21}|^2 < |h_{11}|^2$

Channel Gain Conditions	$P_1^\star(\mathbf{h})$	$P_2^\star(\mathbf{h})$	$\rho^\star(\mathbf{h})$
$\mathcal{R}_1 := \left\{\frac{\|h_{11}\|^2}{\lambda_1} \leq 1, \frac{\|h_{22}\|^2}{\lambda_2} \leq 1\right\}$	0	0	0
$\mathcal{R}_2 := \left\{\frac{\|h_{11}\|^2}{\lambda_1} > 1, \frac{\frac{\|h_{22}\|^2}{\lambda_2}}{1+\frac{\|h_{21}\|^2}{\lambda_1}-\frac{\|h_{21}\|^2}{\|h_{11}\|^2}} \leq 1\right\}$	$\left[\frac{1}{\lambda_1} - \frac{1}{\|h_{11}\|^2}\right]^+$	0	0
$\mathcal{R}_3 := \left\{\frac{\|h_{22}\|^2}{\lambda_2} > 1, \frac{\frac{\|h_{21}\|^2}{\lambda_1}}{\frac{\|h_{22}\|^2}{\lambda_2}} + \frac{\|h_{11}\|^2}{\lambda_1} - \frac{\|h_{21}\|^2}{\lambda_1} \leq 1\right\}$	0	$\left[\frac{1}{\lambda_2} - \frac{1}{\|h_{22}\|^2}\right]^+$	0
$\mathcal{R}_4 := \left\{\frac{\|h_{11}\|^2}{\lambda_1} \leq \frac{\|h_{21}\|^2}{\lambda_1} + \frac{\frac{\|h_{22}\|^2}{\lambda_2}}{\frac{\|h_{21}\|^2}{\lambda_1}+\frac{\|h_{22}\|^2}{\lambda_2}}, \frac{\|h_{21}\|^2}{\lambda_1} + \frac{\|h_{22}\|^2}{\lambda_2} > 1\right\}$	$\frac{\frac{\|h_{21}\|^2}{\lambda_1}+\frac{\|h_{22}\|^2}{\lambda_2}-1}{\left(\frac{\|h_{21}\|^2}{\lambda_1}+\frac{\|h_{22}\|^2}{\lambda_2}\right)^2} \frac{\|h_{21}\|^2}{\lambda_1^2}$	$\frac{\frac{\|h_{21}\|^2}{\lambda_1}+\frac{\|h_{22}\|^2}{\lambda_2}-1}{\left(\frac{\|h_{21}\|^2}{\lambda_1}+\frac{\|h_{22}\|^2}{\lambda_2}\right)^2} \frac{\|h_{22}\|^2}{\lambda_1^2}$	1
$\mathcal{R}_5 := \left\{\left(\mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3 \cup \mathcal{R}_4\right)^{\mathrm{c}}\right\}$	numerically	numerically	numerically

was derived in closed form for certain full-column-rank channel matrices in [23], while in [24] conditions for optimality of beam-forming under PerPC were derived together with an algorithm for optimal multi-stream beam-forming.

Remark 4: The capacity of the fading MISO point-to-point channel with PerPC can not be deduced from capacity results for the fading SISO MAC [25]. Although one could think of a user in the fading SISO MAC as an antenna in the fading MISO point-to-point channel, the analogy stops there; the reason is that in the fading SISO MAC the users send independent inputs while in the fading MISO point-to-point channel the signals sent by the antennas can be correlated. As a result, the sum-capacity achieving power policy for the fading SISO MAC is the following water-filling solution

$$P_i^\star(\mathbf{h}) = \left[\frac{1}{\lambda_i} - \frac{1}{|h_i|^2}\right]^+ \quad \text{if} \quad \frac{|h_i|^2}{\lambda_i} = \max_{k=1,\ldots,K}\left\{\frac{|h_k|^2}{\lambda_k}\right\}. \tag{10}$$

which does not correspond to (7)—for example, under (7) either all the antennas send with a strictly positive power or all stay silent, while under (10) at most one user/antenna sends at any give time.

B. Fading Cognitive Interference Channel

The sum-capacity of the EGCIFC in (4) involves a maximization over the power allocation policy of both transmitters and a correlation coefficient between the inputs over different fading states. We now seek to solve this optimization problem, which in turn depends on the relative strengths of the channel gains between the transmitters and receivers in the channel. We have the following theorems that describe the optimal solution.

Theorem 3 (Strong Interference at the Cognitive Receiver/Rx$_2$): When $|h_{21}|^2 \geq |h_{11}|^2$, the optimal power allocation policy for the EGCIFC in Theorem 1 corresponds to that of point-to-point MISO with PerPC in Theorem 2.

Proof: Given that $|h_{21}|^2 \geq |h_{11}|^2$ is satisfied, then it is clear that $\rho^\star(\mathbf{h}) = 1$ is optimal in (4). We are then left with solving for the optimal power allocation. Setting $\rho^\star(\mathbf{h}) = 1$ reduces the optimization problem in (4) to that in (6) and hence the optimal power allocation strategy is given by Theorem 2. Therefore, setting $R_1 = 0$ turns out to be optimal for the EGCIFC in this regime, i.e., the best use of cognitive user's ability is to broadcast the primary's message. ■

Theorem 4 (Weak Interference at the Cognitive Receiver/Rx$_2$): When $|h_{21}|^2 < |h_{11}|^2$, the optimal power allocation policy for the EGCIFC is summarized in Table I and is one of either of the following policies: (1) both users refrain from transmitting, (2) cognitive transmitter water-fills over $|h_{11}|$, (3) primary transmitter water-fills over $|h_{22}|$, (4) MISO with SumPC type of power allocation, or (5) both users transmit to their intended receivers with non-zero powers (in this case the optimal policy must be determined numerically).

Proof: The proof, based on solving the Lagrangian dual problem of (4), is provided in Appendix B.

One may interpret the policies in Table I as the following. For $\mathcal{R}_1$ both direct link channel gains are "weak" (smaller than the corresponding optimal Lagrange multiplier) and so the optimal scheme for both transmitters is to refrain from allocating power, saving the power for better channel states. In $\mathcal{R}_2$ the cognitive transmitter water-fills over its direct link $|h_{11}|$ while the primary user refrains from allocating any power because in this regime $|h_{11}|$ is "not weak" while $|h_{22}|$ is "weak". One can interpret $\mathcal{R}_3$ similarly to $\mathcal{R}_2$ but with the roles of the users swapped.

The channel gain condition in $\mathcal{R}_4$ implies that $\frac{|h_{11}|^2}{\lambda_1} \leq \frac{|h_{21}|^2}{\lambda_1} + \frac{|h_{22}|^2}{\lambda_2}$ (given that $\frac{|h_{21}|^2}{\lambda_1} + \frac{|h_{22}|^2}{\lambda_2} > 1$); in this case the sum of the channel gains to the primary receiver is "stronger" than that of the direct gain to the cognitive receiver and performing a point-to-point MISO-type power allocation is optimal. In $\mathcal{R}_5$ both transmitters send with non-zero power; in this case a closed-form solution for the optimal power allocation policy is not available. ■

Remark 5: The above analysis showed that a separable achievable scheme is sum-capacity optimal. In order to characterize the whole capacity region one needs bounds on R_1 and R_2 too; the cut-set approach gives such bounds. Therefore the capacity region of the EGCIFC is outer bounded by

$$R_1 - \epsilon_N \leq \frac{1}{N}\sum_{i=1}^{N} I(X_{1Gi}; Y_{1i}|X_{2Gi}, \mathbf{h}_i)$$

$$R_2 - \epsilon_N \leq \frac{1}{N}\sum_{i=1}^{N} I(Y_{2Gi}; X_{1i}, X_{2Gi}|\mathbf{h}_i)$$

$$R_1 + R_2 - 2\epsilon_N \leq \frac{1}{N}\sum_{i=1}^{N} I(Y_{2i}; X_{1Gi}, X_{2Gi}|\mathbf{h}_i) + I(X_{1Gi}; Y_{1i}|X_{2Gi}, Y'_{2i}, \mathbf{h}_i),$$

where the last bound is from Theorem 1 and the single rate bounds are cut-set bounds, similarly to [19, eq. (8)]. As for the sum-capacity, the whole region is exhausted by considering jointly Gaussian inputs and where the region can be tightened by choosing any $Y_2' \sim Y_2$. By further taking the appropriate limit over N as done in [3] and by considering the input covariance as in (3), the upper bound region can be expressed as

$$R_1 \le \mathbb{E}\left[\log\left(1 + |h_{11}|^2 P_1(\mathbf{h})\left(1 - |\rho(\mathbf{h})|^2\right)\right)\right], \tag{11a}$$

$$R_2 \le \mathbb{E}\left[\log\left(1 + \mathbf{h}_2 \Sigma(\mathbf{h}) \mathbf{h}_2^\dagger\right)\right] \tag{11b}$$

$$R_1 + R_2 \le \mathbb{E}\Bigg[\log\left(1 + \mathbf{h}_2 \Sigma(\mathbf{h}) \mathbf{h}_2^\dagger\right) + \log\left(\frac{1 + \left(1 - |\rho(\mathbf{h})|^2\right)\max\{|h_{11}|^2, |h_{21}|^2\} P_1(\mathbf{h})}{1 + \left(1 - |\rho(\mathbf{h})|^2\right)|h_{21}|^2 P_1(\mathbf{h})}\right)\Bigg], \tag{11c}$$

By considering the achievable scheme in [19, eqs. (22) and (23)], which was shown to be at most to within 1 bit per channel use per user of the capacity region outer bound for the static/non-fading case, the outer bound in (11) can be shown to be achievable to within 1 bit per channel use per user as well as in the EGCIFC. We note that this achievable scheme involves two DPC steps, one per user (while the scheme in (5) that only has one DPC step) and it is only approximately optimal to within a constant gap (while the scheme in (5) is exactly sum-rate optimal).

We therefore conclude that, in order to characterize the whole capacity region of the EGCIFC to within a gap, it suffices to characterize the closure of the outer bound in (11), which can be done by solving the following family of convex optimization problems: for each $\lambda \in [0, 1]$

$$C_{\text{EGCIFC,region}}(\lambda) := \max\{\lambda R_1 + (1 - \lambda) R_2\}, \tag{12}$$

where the maximization is over the rate pairs (R_1, R_2) in (11). Solving the optimization problem in (12) is not a trivial extension of the sum-capacity results presented in Theorems 3 and 4 and is beyond the scope of this paper. This is so because one needs to consider which bounds are active in (11) in order to determine the optimal 'corner point' as a function of $\lambda \in [0, 1]$; the coordinate of such a point must be plugged in the optimization problem in (12) and the corresponding KKT conditions must be worked out similarly to Appendix B. We expect that there will be regimes in which the KKT conditions must be solved numerically as for the sum-capacity.

IV. Numerical Results

In this section we numerically evaluate Theorems 1 and 2 for the case where the channel gains are independent Rayleigh random variables, not necessarily with the same mean parameter.

A. The Point-to-Point MISO Channel With PerPC

We first consider a 2×1 point-to-point MISO channel. The channel vector $[h_1, h_2]$ at each channel use has independent

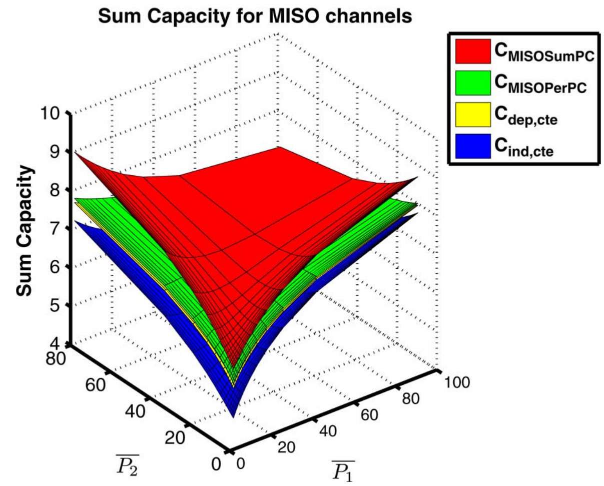

Fig. 2. The capacity of the point-to-point MISO channel with PerPC (surface in green, from (13)) is upper bounded by that of a MISO channel with SumPC (surface in red, from (16)) and lower bounded by that of a MISO channel with constant power allocation and dependent inputs (surface in yellow, from (14)) and with constant power allocation and independent inputs (surface in blue, from (15)).

and exponentially distributed components with means $\gamma_1 = \mathbb{E}[|h_1|^2] = 5$ and $\gamma_2 = \mathbb{E}[|h_2|^2] = 2$. The transmit antennas are subject to the average power constraints $\overline{P}_1$ and $\overline{P}_2$. In Fig. 2 four surfaces representing the capacities of different MISO channels are plotted as a function of the average transmit antenna power constraints $\overline{P}_1$ and $\overline{P}_2$. The surfaces corre spond to:

1) the MISO channel with PerPC

$$C_{\text{MISOPerPC}} = \mathbb{E}\left[\log\left(1 + \left(\sqrt{|h_1|^2 P_1^\star(\mathbf{h})} + \sqrt{|h_2|^2 P_2^\star(\mathbf{h})}\right)^2\right)\right] \tag{13}$$

where $P_1^\star(\mathbf{h})$ and $P_2^\star(\mathbf{h})$ are given in (7) for $j \in [1:2]$;

2) the MISO channel with constant power allocation and beam-forming (where the instantaneous phases are known to the transmitters to allow for coherent beam-forming)

$$C_{\text{dep,cte}} = \mathbb{E}\left[\log\left(1 + \left(\sqrt{|h_1|^2 \overline{P}_1} + \sqrt{|h_2|^2 \overline{P}_2}\right)^2\right)\right]; \tag{14}$$

3) the MISO channel with constant power allocation and independent signaling (instantaneous phases are not known at the transmitter and they are independent and uniformly distributed in $[0, 2\pi]$ as in [15])

$$C_{\text{indep,cte}} = \mathbb{E}\left[\log\left(1 + |h_1|^2 \overline{P}_1 + |h_2|^2 \overline{P}_2\right)\right]; \tag{15}$$

and 4) the MISO channel with a SumPC

$$C_{\text{MISOSumPC}} = \mathbb{E}\left[\log\left(1 + \left(\sqrt{|h_1|^2 P_1^\star(\mathbf{h})} + \sqrt{|h_2|^2 P_2^\star(\mathbf{h})}\right)^2\right)\right] \tag{16}$$

where $P_1^\star(\mathbf{h})$ and $P_2^\star(\mathbf{h})$ are given in (9) for $i \in [1:2]$.

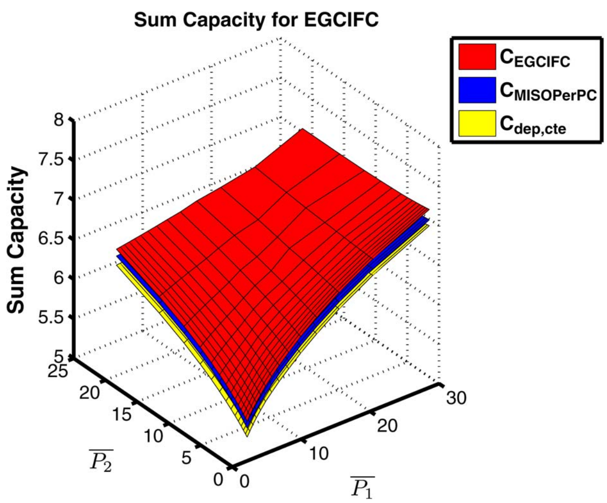

Fig. 3. The sum-capacity of the EGCIFC which is skewed to be in strong interference with high probability (surface in red) is plotted with the sum-capacity achieved when considering a MISO achievability with PerPC (surface in blue). The MISO power allocation is almost sum-capacity achieving as the two surfaces almost overlap. The constant power allocation scheme with dependent inputs (surface in yellow) is again a lower bound on the sum-capacity of EGCIFC. Note that perfect CSI at both transmitters is needed for coherent beam-forming.

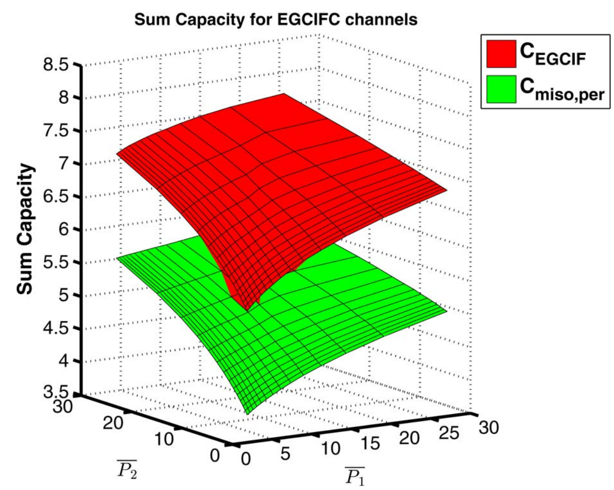

Fig. 4. The sum-capacity of EGCIFC which is skewed to be in weak interference with high probability (surface in green) is plotted with the sum-capacity achieved when considering a MISO achievability scheme with PerPC. The power allocation as in the MISO with PerPC (surface in green) is not optimal as in the case when the channel is skewed to be in strong interference.

Numerical evaluations show that the capacity of the point-to-point MISO with PerPC is upper and lower bounded by that of the MISO with SumPC and that of constant power allocation respectively. Also as expected, dependent constant inputs in (14) outperform independent constant inputs in (15).

Remark 6: In [15] the point-to-point MISO channel with PerPC with Rayleigh fading with no CSI at the transmitters was compared to that with SumPC and with independent signaling; the author noted that $C_{\text{MISOPerPC}} = C_{\text{indep,cte}}$ because of the phase being independent and uniformly distributed in $[0, 2\pi]$. Since here we assume the transmitter has CSI we have $C_{\text{MISOPerPC}} \geq C_{\text{indep,cte}}$.

Remark 7: In the case of dependent inputs and beamforming, the channel is assumed to have CSI at the transmitter to account for the channel gain phases and the ability to coherently beam-form. This explains why $C_{\text{MISOPerPC}}$ is almost the same as $C_{\text{dep,cte}}$, which may have practical implications.

B. The EGCIFC Sum-Capacity

We now consider two different scenarios for the EGCIFC corresponding again to Rayleigh fading channels with different means: Case 1) the means are chosen such that the channel experiences strong interference with high probability, and Case 2) the means are chosen to experience weak interference with high probability. Monte Carlo simulations were used to evaluate the capacities.

In Fig. 3 the sum-capacity for the EGCIFC having $|h_{11}|^2, |h_{21}|^2$ and $|h_{22}|^2$ exponentially distributed with $\gamma_0 = \mathbb{E}[|h_{11}|^2] = 1$, $\gamma_1 = \mathbb{E}[|h_{21}|^2] = 5$ and $\gamma_2 = \mathbb{E}[|h_{22}|^2] = 2$ (skewed with high probability to be in strong interference since $\mathbb{P}[|h_{21}|^2 \geq |h_{11}|^2] = \frac{\gamma_1}{\gamma_1+\gamma_0} = \frac{5}{6}$) is plotted along with the sum-capacity of the system using the MISO with PerPC transmit strategy. The latter is not optimal in general when the channel experiences weak interference states. Considerations similar to those made in Remark 6 may be made here; moreover the surface representing the sum-capacity $C_{\text{MISOPerPC}}$ approaches that of C_{EGCIFC} (as expected since we are skewed to be in strong interference where the scheme corresponding to a MISO channel with PerPC is optimal).

In Fig. 4 the sum-capacity for the EGCIFC having mean parameters $\gamma_0 = 5$, $\gamma_1 = 1$ and $\gamma_2 = 2$ is plotted (skewed with high probability to be in weak interference $\mathbb{P}[|h_{21}|^2 < |h_{11}|^2] = \frac{5}{6}$) along with the sum-capacity achieved by using the power allocation corresponding to that of a MISO channel with PerPC. As expected, the MISO scheme with PerPC (not optimal for weak interference) does not perform as well as in the regime where the channel is skewed to be in strong interference with high probability.

In Fig. 5 we choose the channel gains to be identically distributed with mean parameter $\gamma_0 = \gamma_1 = \gamma_2 = 1$ and plot the sum-capacity of the EGCIFC and an achievability scheme corresponding to constant power allocation, but with the optimal correlation coefficient which changes with each fading state, i.e., the solution of

$$C_{\text{sum}} = \mathbb{E}\Bigg[\max_{|\rho(\mathbf{h})|\leq 1} \log\left(1+|h_{21}|^2\overline{P_1}+|h_{22}|^2\overline{P_2}+2|\rho(\mathbf{h})|\sqrt{|h_{21}|^2\overline{P_1}\,|h_{22}|^2\overline{P_2}}\right) + \log\left(\frac{1+(1-|\rho(\mathbf{h})|^2)\max\{|h_{11}|^2,|h_{21}|^2\}\overline{P_1}}{1+(1-|\rho(\mathbf{h})|^2)\,|h_{21}|^2\overline{P_1}}\right)\Bigg].$$

This is solved for the optimal correlation coefficient numerically. It is interesting to note that by optimizing the correlation coefficient only (and not the power allocation, i.e., keeping the power constant) one can approach (a difference of around 0.23 bits/channel use), at least for these channel conditions, the sum-capacity of the EGCIFC. This may have implications in practice—i.e., constant power may be good enough if one optimally finds the correlation coefficient. We note however that to obtain the optimal correlation coefficient at each fading state, full CSI is still required at both transmitters.

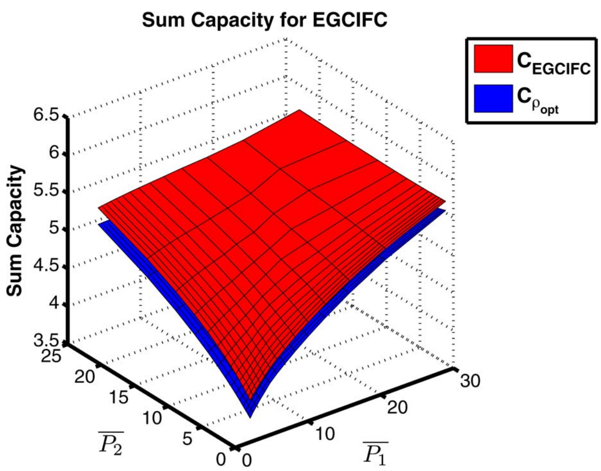

Fig. 5. Two surfaces representing the sum-capacity of EGCIFC with the optimal power allocation (red) and the sum-capacity while considering constant power allocation and an optimized correlation coefficient at each fading state (blue). Although the optimal power allocation was not utilized, the blue surface is almost capacity achieving.

V. Conclusion

In this work we characterized the sum-capacity of the ergodic fading Gaussian cognitive interference channel. A separable scheme (power allocation depends only on the current fading state and coding need not be done across fading states) was shown to be optimal. This is in contrast to the classical interference channel, which is not separable, but similar to the fading broadcast channel and a one-sided fading interference channel. The optimal power policy in strong interference was shown to be that corresponding to a MISO point-to-point channel with per-antenna power constraints. As a side result of independent interest, we derived in closed-form the ergodic capacity of this channel model. Numerical results show that, at least for certain channel parameters, optimizing the correlation coefficient between the inputs of the two users, while keeping the powers fixed, performs nearly as well as optimizing both the powers and correlation coefficient. We discussed how the present work can be extended to the characterization of the whole region, which is the subject of current investigation. Extensions to settings with an arbitrary number of users is also an interesting direction for future work.

Appendix

A. Proof of Theorem 2

The point-to-point MISO capacity with PerPC and n transmit antennas is the solution of the following optimization problem:

$$\max_{P_i(\mathbf{h})\geq 0:\mathbb{E}[P_i(\mathbf{h})]\leq \overline{P}_i, i\in[1:n]} \mathbb{E}\left[\log\left(1+\left(\sum_i |h_i|\sqrt{P_i(\mathbf{h})}\right)^2\right)\right].$$

The Lagrange dual problem, for $\lambda_i \geq 0, i \in [1:n]$, is

$$\mathcal{L}=\mathbb{E}\left[\log\left(1+\left(\sum_i |h_i|\sqrt{P_i(\mathbf{h})}\right)^2\right)-\sum_i \lambda_i\left(P_i(\mathbf{h})-\overline{P}_i\right)\right].$$

By taking partial derivatives of $\mathcal{L}$ we obtain for every $i \in [1:n]$

$$\frac{\partial \mathcal{L}}{\partial P_i(\mathbf{h})} = \frac{\theta}{1+\theta^2}\frac{|h_i|}{\sqrt{P_i}} - \lambda_i = 0, \qquad \theta := \sum_i |h_i|\sqrt{P_i(\mathbf{h})},$$

$$\iff \sqrt{P_i(\mathbf{h})} = \frac{\theta}{1+\theta^2}\frac{|h_i|}{\lambda_i}, \tag{17}$$

which implies

$$\theta = \sum_i |h_i|\sqrt{P_i(\mathbf{h})} = \frac{\theta}{1+\theta^2}\sum_i \frac{|h_i|^2}{\lambda_i} \tag{18}$$

$$\iff 1+\theta^2 = \sum_i \frac{|h_i|^2}{\lambda_i} \geq 1 \tag{19}$$

$$\iff \theta = \sqrt{\left(\sum_i \frac{|h_i|^2}{\lambda_i} - 1\right)^+}. \tag{20}$$

From (17), the optimal power for antenna $i \in [1:n]$ becomes

$$\sqrt{P_j(\mathbf{h})} = \frac{\sqrt{\left(\sum_i |h_i|^2\lambda_i - 1\right)^+}}{\sum_i \frac{|h_i|^2}{\lambda_i}}\frac{|h_j|}{\lambda_j} \tag{21}$$

$$\iff \lambda_j P_j(\mathbf{h}) = \left(1-\frac{1}{\sum_i \frac{|h_i|^2}{\lambda_i}}\right)^+ \frac{\frac{|h_j|^2}{\lambda_j}}{\sum_i \frac{|h_i|^2}{\lambda_i}}, \tag{22}$$

and thus the capacity is

$$\mathbb{E}\left[\log\left(1+\left[\sum_i \frac{|h_i|^2}{\lambda_i}-1\right]^+\right)\right] = \mathbb{E}\left[\log^+\left(\sum_i \frac{|h_i|^2}{\lambda_i}\right)\right],$$

and the Lagrange multipliers (from (22)) solve the non-linear system of equations

$$\lambda_j \overline{P_j} = \mathbb{E}\left[\left[1-\frac{1}{\sum_i \frac{|h_i|^2}{\lambda_i}}\right]^+ \frac{1}{\sum_i \frac{|h_i|^2}{\lambda_i}}\frac{|h_j|^2}{\lambda_j}\right]. \tag{23}$$

B. Proof of Theorem 4

The sum-capacity of the EGCIFC is given in (4). When $|h_{21}|^2 \geq |h_{11}|^2$, the sum-capacity in (4) is clearly maximized by $|\rho^\star(\mathbf{h})| = 1$ and thus we must solve

$$\max_{P_i\geq 0:\mathbb{E}[P_i(\mathbf{h})]\leq \overline{P}_i, i\in[1:2]} \mathbb{E}\left[\log\left(1+\left(\sum_i |h_{2i}|\sqrt{P_i(\mathbf{h})}\right)^2\right)\right]. \tag{24}$$

The problem in (24) is that of finding the optimal power allocation for a point-to-point MISO with PerPC and the solution was presented in Appendix A.

When $|h_{11}|^2 > |h_{21}|^2$, (25)

the Lagrangian dual of the sum-capacity in (4) is

$$\begin{aligned}\mathcal{L} =\mathbb{E}\Bigg[&\log\left(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^{\dagger}\right)\\ &+\log\left(\frac{1+\left(1-|\rho(\mathbf{h})|^2\right)|h_{11}|^2P_1(\mathbf{h})}{1+(1-|\rho(\mathbf{h})|^2)|h_{21}|^2P_1(\mathbf{h})}\right)-\lambda_1\left(P_1(\mathbf{h})-\bar{P}_1\right)\\ &-\lambda_2\left(P_2(\mathbf{h})-\bar{P}_2\right)+\mu_1(\mathbf{h})P_1(\mathbf{h})+\mu_2(\mathbf{h})P_2(\mathbf{h})\\ &+\gamma_1(\mathbf{h})\,|\rho(\mathbf{h})|-\gamma_2(\mathbf{h})\,(1-|\rho(\mathbf{h})|)\Bigg].\end{aligned}$$

whose KKT conditions are

$$\mu_i(\mathbf{h})P_i(\mathbf{h})=0,\ i\in[1,2], \tag{26a}$$
$$\gamma_1(\mathbf{h})|\rho(\mathbf{h})|=0, \tag{26b}$$
$$\gamma_2(\mathbf{h})\,(1-|\rho(\mathbf{h})|)=0, \tag{26c}$$
$$P_i(\mathbf{h})\geq 0,\ i\in[1,2], \tag{26d}$$
$$|\rho(\mathbf{h})|\leq 1, \tag{26e}$$
$$|\rho(\mathbf{h})|\geq 0, \tag{26f}$$
$$\gamma_i(\mathbf{h}),\mu_i(\mathbf{h}),\lambda_i\geq 0,\ i\in[1,2], \tag{26g}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial P_1(\mathbf{h})} &= \frac{|h_{21}|^2+|\rho(\mathbf{h})|\sqrt{|h_{21}|^2|h_{22}|^2}\sqrt{\frac{P_2(\mathbf{h})}{P_1(\mathbf{h})}}}{\left(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^{\dagger}\right)}\\ &+\frac{\left(|h_{11}|^2-|h_{21}|^2\right)\ \left(1-|\rho(\mathbf{h})|^2\right)}{(1+(1-|\rho(\mathbf{h})|^2)|h_{21}|^2P_1(\mathbf{h}))\left(1+\left(1-|\rho(\mathbf{h})|^2\right)|h_{11}|^2P_1(\mathbf{h})\right)}\\ &-(\lambda_1-\mu_1(\mathbf{h}))=0,\end{aligned} \tag{26h}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial P_2(\mathbf{h})} &= \frac{|h_{22}|^2+|\rho(\mathbf{h})|\sqrt{|h_{21}|^2|h_{22}|^2}\sqrt{\frac{P_1(\mathbf{h})}{P_2(\mathbf{h})}}}{\left(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^{\dagger}\right)}\\ &-(\lambda_2-\mu_2(\mathbf{h}))=0,\end{aligned} \tag{26i}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\,|\rho(\mathbf{h})|} &= \frac{\sqrt{|h_{21}|^2P_1(\mathbf{h})|h_{22}|^2P_2(\mathbf{h})}}{\left(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^{\dagger}\right)}\\ &-\frac{\left(|h_{11}|^2-|h_{21}|^2\right)|\rho(\mathbf{h})|P_1(\mathbf{h})}{\left(1+\left(1-|\rho(\mathbf{h})|^2\right)|h_{21}|^2P_1(\mathbf{h})\right)\left(1+\left(1-|\rho(\mathbf{h})|^2\right)|h_{11}|^2P_1(\mathbf{h})\right)}\\ &+(\gamma_1(\mathbf{h})-\gamma_2(\mathbf{h}))=0.\end{aligned} \tag{26j}$$

We next analyze different possible solutions.

1) On the Optimality of $|\rho(\mathbf{h})|=0$ (i.e., Independent Inputs): If $|\rho^{\star}(\mathbf{h})|=0$ is an optimal solution then $\gamma_1(\mathbf{h})\geq 0$ and $\gamma_2(\mathbf{h})=0$ (from (26b), (26c) and (26g)) and the following KKT conditions must hold

$$\begin{aligned}\text{From (26h)}: &\frac{|h_{21}|^2}{1+|h_{21}|^2P_1(\mathbf{h})+|h_{22}|^2P_2(\mathbf{h})}\\ &+\frac{|h_{11}|^2-|h_{21}|^2}{(1+|h_{21}|^2P_1(\mathbf{h}))\,(1+|h_{11}|^2P_1(\mathbf{h}))}=\lambda_1-\mu_1(\mathbf{h}),\end{aligned} \tag{27a}$$

$$\text{From (26i)}: \frac{|h_{22}|^2}{1+|h_{21}|^2P_1(\mathbf{h})+|h_{22}|^2P_2(\mathbf{h})}=\lambda_2-\mu_2(\mathbf{h}), \tag{27b}$$

$$\text{From (26j)}: \frac{\sqrt{|h_{21}|^2P_1(\mathbf{h})|h_{22}|^2P_2(\mathbf{h})}}{1+|h_{21}|^2P_1(\mathbf{h})+|h_{22}|^2P_2(\mathbf{h})}=-\gamma_1(\mathbf{h})\leq 0. \tag{27c}$$

From (27c) we have that $P_1(\mathbf{h})\cdot P_2(\mathbf{h})=0$, that is, the powers cannot be simultaneously strictly positive. We now proceed by finding the optimal power allocation.

Subcase B1.1 $P_1(\mathbf{h})=0$ and $P_2(\mathbf{h})=0$:

From (26a) :

$$P_1(\mathbf{h})=0\rightarrow\mu_1(\mathbf{h})\geq 0 \text{ and } P_2(\mathbf{h})=0\rightarrow\mu_2(\mathbf{h})\geq 0, \tag{28a}$$

$$\text{From (27b)}: |h_{22}|^2=\lambda_2-\mu_2(\mathbf{h})\leq\lambda_2\rightarrow\frac{|h_{22}|^2}{\lambda_2}\leq 1, \tag{28b}$$

$$\text{From (27a)}: |h_{11}|^2=\lambda_1-\mu_1(\mathbf{h})\leq\lambda_1\rightarrow\frac{|h_{11}|^2}{\lambda_1}\leq 1. \tag{28c}$$

We therefore conclude that $(P_1^{\star}(\mathbf{h}),P_2^{\star}(\mathbf{h}),|\rho^{\star}(\mathbf{h})|)=(0,0,0)$ is optimal when

$$\mathcal{R}_1:=\left\{\text{eq.(25) holds, }\frac{|h_{11}|^2}{\lambda_1}\leq 1,\frac{|h_{22}|^2}{\lambda_2}\leq 1\right\}.$$

Subcase B1.2 $P_1(\mathbf{h})>0$ and $P_2(\mathbf{h})=0$:

From (26a) :

$$P_1(\mathbf{h})>0\rightarrow\mu_1(\mathbf{h})=0 \text{ and } P_2(\mathbf{h})>0\rightarrow\mu_2(\mathbf{h})\geq 0, \tag{29a}$$

From (27a) :

$$\begin{aligned}&\frac{|h_{21}|^2}{1+|h_{21}|^2P_1(\mathbf{h})}+\frac{|h_{11}|^2-|h_{21}|^2}{(1+|h_{21}|^2P_1(\mathbf{h}))\,(1+|h_{11}|^2P_1(\mathbf{h}))}=\lambda_1\\ &\rightarrow P_1(\mathbf{h})=\frac{1}{\lambda_1}-\frac{1}{|h_{11}|^2}=:P_{1a}(\mathbf{h}),\end{aligned} \tag{29b}$$

$$\begin{aligned}\text{From (27b)}: &\frac{|h_{22}|^2}{1+|h_{21}|^2P_1(\mathbf{h})}=\lambda_2-\mu_2(\mathbf{h})\\ &\rightarrow P_1(\mathbf{h})=\frac{|h_{22}|^2}{(\lambda_2-\mu_2(\mathbf{h}))\,|h_{21}|^2}-\frac{1}{|h_{21}|^2}=:P_{1b}(\mathbf{h}).\end{aligned} \tag{29c}$$

By imposing $P_{1a}(\mathbf{h})=P_{1b}(\mathbf{h})$ and since $\mu_2(\mathbf{h})\geq 0$, we obtain $1+\frac{\lambda_1}{|h_{21}|^2}\geq\frac{\lambda_1}{|h_{11}|^2}+\frac{\lambda_1|h_{22}|^2}{\lambda_2|h_{21}|^2}$. We therefore conclude that $(P_1^{\star}(\mathbf{h}),P_2^{\star}(\mathbf{h}),|\rho^{\star}(\mathbf{h})|)=\left(\left[\frac{1}{\lambda_1}-\frac{1}{|h_{11}|^2}\right]^+,0,0\right)$ is optimal when

$$\mathcal{R}_2:=\left\{\text{eq.(25) holds, }\frac{|h_{11}|^2}{\lambda_1}>1,\frac{\frac{|h_{22}|^2}{\lambda_2}}{1+\frac{|h_{21}|^2}{\lambda_1}-\frac{|h_{21}|^2}{|h_{11}|^2}}\leq 1\right\}.$$

Subcase B1.3 $P_1(\mathbf{h})=0$ and $P_2(\mathbf{h})>0$:

From (26a) :

$$P_1(\mathbf{h})=0\rightarrow\mu_1(\mathbf{h})\geq 0 \text{ and } P_2(\mathbf{h})>0\rightarrow\mu_2(\mathbf{h})=0,$$

$$\begin{aligned}\text{From (27a)}: &\frac{|h_{21}|^2}{1+|h_{22}|^2P_2(\mathbf{h})}+\left(|h_{11}|^2-|h_{21}|^2\right)\\ &=\lambda_1-\mu_1(\mathbf{h})\leq\lambda_1,\end{aligned} \tag{30a}$$

$$\begin{aligned}\text{From (27b)}: &\frac{|h_{22}|^2}{1+|h_{22}|^2P_2(\mathbf{h})}=\lambda_2\\ &\rightarrow P_2^{\star}(\mathbf{h})=\frac{1}{\lambda_2}-\frac{1}{|h_{22}|^2}>0.\end{aligned} \tag{30b}$$

By evaluating (30a) for $P_2^\star(\mathbf{h})$ in (30b) gives $|h_{21}|^2\Big(-1+\frac{\lambda_2}{|h_{22}|^2}\Big)+|h_{11}|^2\le\lambda_1$. We therefore conclude that $(P_1^\star(\mathbf{h}), P_2^\star(\mathbf{h}), |\rho^\star(\mathbf{h})|)=\left(0,\left[\frac{1}{\lambda_2}-\frac{1}{|h_{22}|^2}\right]^+,0\right)$ is optimal when

$$\mathcal{R}_3:=\Bigg\{\text{eq.(25) holds},\ \frac{|h_{22}|^2}{\lambda_2}>1,\ \frac{\lambda_2|h_{21}|^2}{\lambda_1|h_{22}|^2}+\frac{|h_{11}|^2}{\lambda_1}-\frac{|h_{21}|^2}{\lambda_1}\le 1\Bigg\}.$$

2) On the Optimality of $|\rho^\star(\mathbf{h})|=1$ (i.e., Identical Inputs up to Affine Transformation): If $|\rho(\mathbf{h})|=1$ is an optimal solution then $\gamma_1(\mathbf{h})=0$ and $\gamma_2(\mathbf{h})\ge 0$ (from (26b), (26c) and (26g)) and the following KKT conditions must hold. From (26h):

$$\frac{|h_{21}|^2+\sqrt{|h_{21}|^2|h_{22}|^2}\sqrt{\frac{P_2(\mathbf{h})}{P_1(\mathbf{h})}}}{\Big(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^\dagger\Big)}=\lambda_1-\mu_1(\mathbf{h}), \tag{31a}$$

$$\text{From (26i)}:\ \frac{|h_{22}|^2+\sqrt{|h_{21}|^2|h_{22}|^2}\sqrt{\frac{P_1(\mathbf{h})}{P_2(\mathbf{h})}}}{\Big(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^\dagger\Big)}=\lambda_2-\mu_2(\mathbf{h}), \tag{31b}$$

$$\text{From (26j)}:\ \frac{\sqrt{|h_{21}|^2P_1(\mathbf{h})|h_{22}|^2P_2(\mathbf{h})}}{\Big(1+\mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2^\dagger\Big)} \ge \left(|h_{11}|^2-|h_{21}|^2\right)P_1(\mathbf{h}). \tag{31c}$$

If $P_1(\mathbf{h})\cdot P_2(\mathbf{h})=0$ in (4) then trivially $|\rho^\star(\mathbf{h})|=0$ is optimal (as in the previous case) and thus under the assumption that $|\rho^\star(\mathbf{h})|=1$ we only need to consider $P_1(\mathbf{h})\cdot P_2(\mathbf{h})>0$. If both powers are strictly positive we further have that $\mu_1(\mathbf{h})=\mu_2(\mathbf{h})=0$ and thus the system of equations in (31) is equivalent to

$$\xi:=\frac{\sqrt{|h_{21}|^2P_1(\mathbf{h})}+\sqrt{|h_{22}|^2P_2(\mathbf{h})}}{1+(\sqrt{|h_{21}|^2P_1(\mathbf{h})}+\sqrt{|h_{22}|^2P_2(\mathbf{h})})^2},$$

$$\text{From (31a)}:\ |h_{21}|\,\xi=\lambda_1\sqrt{P_1(\mathbf{h})}, \tag{32a}$$

$$\text{From (31b)}:\ |h_{22}|\,\xi=\lambda_2\sqrt{P_2(\mathbf{h})}, \tag{32b}$$

$$\text{From (31c)}:\ \frac{\sqrt{|h_{21}|^2P_1(\mathbf{h})\,|h_{22}|^2P_2(\mathbf{h})}}{\sqrt{|h_{21}|^2P_1(\mathbf{h})}+\sqrt{|h_{22}|^2P_2(\mathbf{h})}}\,\xi \ge \left(\frac{|h_{11}|^2}{\lambda_1}-\frac{|h_{21}|^2}{\lambda_1}\right)\lambda_1P_1(\mathbf{h}). \tag{32c}$$

This is formally the same optimization problem as that for the point-to-point MISO with PerPC in (17) and thus the optimal powers are given by

$$P_1^\star(\mathbf{h})=\frac{\frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}-1}{\left(\frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}\right)^2}\,\frac{|h_{21}|^2}{\lambda_1^2}>0, \tag{33}$$

$$P_2^\star(\mathbf{h})=\frac{\frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}-1}{\left(\frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}\right)^2}\,\frac{|h_{22}|^2}{\lambda_1^2}>0. \tag{34}$$

We next need to verify that, for these optimizing powers, we satisfy (32c); by doing so, we conclude that $(P_1^\star(\mathbf{h}), P_2^\star(\mathbf{h}), |\rho^\star(\mathbf{h})|)=(\text{eq.}(33),\text{eq.}(34),1)$ is optimal when

$$\mathcal{R}_4:=\Bigg\{\text{eq.(25) holds},\ \frac{|h_{11}|^2}{\lambda_1}\le\frac{|h_{21}|^2}{\lambda_1}+\frac{\frac{|h_{22}|^2}{\lambda_2}}{\frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}},\ \frac{|h_{21}|^2}{\lambda_1}+\frac{|h_{22}|^2}{\lambda_2}>1\Bigg\}.$$

3) On the Optimality $0<|\rho(\mathbf{h})|<1$: As mentioned earlier, if $P_1(\mathbf{h})\cdot P_2(\mathbf{h})=0$ in (4) then trivially $|\rho^\star(\mathbf{h})|=0$ is optimal and thus under the assumption that $|\rho^\star(\mathbf{h})|>0$ we only need to consider $P_1(\mathbf{h})\cdot P_2(\mathbf{h})>0$.

When $P_1^\star(\mathbf{h})>0$, $P_2^\star(\mathbf{h})>0$ and $0<|\rho^\star(\mathbf{h})|<1$ the system of equations in (26) does not seem to have a closed form solution. Therefore in

$$\mathcal{R}_5:=\{\text{eq.(25) holds},\ (\mathcal{R}_1\cup\mathcal{R}_2\cup\mathcal{R}_3\cup\mathcal{R}_4\cup\mathcal{R}_5)^{\text{c}}\}$$

the optimal power allocation must be found numerically.

References

[1] D. Maamari, D. Tuninetti, and N. Devroye, "The capacity of the ergodic MISO channel with per-antenna power constraint and an application to the fading cognitive interference channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Honolulu, HI, USA, Jun. 2014, pp. 1727–1731.

[2] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[3] L. Sankar, X. Shang, and V. Poor, "Ergodic fading interference channels: Sum-capacity and separability," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2605–2626, May 2011.

[4] R. A. Tannious and A. Nosratinia, "Cognitive radio protocols based on exploiting hybrid ARQ retransmissions," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2833–2841, Sep. 2010.

[5] V. Annapureddy, A. El Gamal, and V. V. Veeravalli, "Degree of freedom of interference channels with CoMP transmission and reception," *IEEE Trans. Inf. Theory*, vol. 58, no. 9, pp. 5740–5760, Sep. 2012.

[6] A. El Gamal and V. V. Veeravalli, "Dynamic interference management," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2013, pp. 1902–1906.

[7] D. Maamari, D. Tuninetti, and N. Devroye, "Approximate sum-capacity of K-user cognitive interference channels with cumulative message sharing," *IEEE J. Select. Areas Commun.*, vol. 32, no. 3, pp. 654–666, Mar. 2014.

[8] D. Tuninetti, "Gaussian fading interference channels: Power control," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Monterrey, CA, USA, Oct. 2008, pp. 701–706.

[9] V. R. Cadambe and S. A. Jafar, "Parallel Gaussian interference channels are not always separable," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3983–3990, Sep. 2009.

[10] D. Tse and S. V. Hanly, "Multiaccess fading channels. I. Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Inf. Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[11] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels—Part I: Ergodic capacity," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.

[12] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[13] X. Kang, Y.-C. Liang, A. Nallanathan, H. K. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[14] R. Zhang, S. Cui, and Y.-C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5161–5178, Nov. 2009.

[15] M. Vu, "MISO capacity with per-antenna power constraint," *IEEE Trans. Commun.*, vol. 59, no. 5, pp. 1268–1274, May 2011.

[16] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.

[17] A. Jovicic and P. Viswanath, "Cognitive radio: An information-theoretic perspective," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3945–3958, Sep. 2009.

[18] S. Rini, D. Tuninetti, and N. Devroye, "New inner and outer bounds for the discrete memoryless cognitive interference channel and some new capacity results," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4087–4109, Jul. 2011.

[19] S. Rini, D. Tuninetti, and N. Devroye, "Inner and outer bounds for the Gaussian cognitive interference channel and new capacity results," *IEEE Trans. Inf. Theory*, vol. 58, no. 2, pp. 820–848, Feb. 2012.

[20] T. Cover and J. Thomas, *Elements of Information Theory: Second Edition*. Hoboken, NJ, USA: Wiley, 2006.

[21] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, no. 3, pp. 439–441, May 1983.

[22] Y. Zhu and M. Vu, "Iterative mode-dropping for the sum capacity of MIMO-MAC with per-antenna power constraint," *IEEE Trans. Commun.*, vol. 60, no. 9, pp. 2421–2426, Sep. 2012.

[23] D. Tuninetti, "On the capacity of the AWGN MIMO channel under per-antenna power constraints," in *Proc. IEEE Int. Conf. Commun.*, Sydney, Australia, Jun. 2014, pp. 2153–2157.

[24] Z. Pi, "Optimal transmitter beamforming with per-antenna power constraints," in *Proc. IEEE Int. Conf. Commun.*, Ottawa, ON, Canada, Jun. 2012, pp. 3779–3784.

[25] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE Int. Conf. Commun.*, Seattle, WA, USA, Jun. 1995, vol. 1, pp. 331–335.

Diana Maamari received the Bachelor of Science degree (with honors) in electrical engineering and the Master of Science degree (with honors) in electrical engineering from the University of Balamand, Al Koura, Lebanon, in 2009 and 2011, respectively. She is currently working toward the Ph.D. degree with the University of Illinois at Chicago, IL, USA. Her current research focuses on multi-user information theory and its applications to cognitive radio channels.

Natasha Devroye received the B.Eng. (Hons.) degree in electrical engineering from McGill University, Montreal, QC, Canada, in 2001 and the M.Sc and Ph.D. degrees in engineering sciences from Harvard University, Cambridge, MA, USA, in 2003 and 2007, respectively. She has been an Assistant Professor with the Department of Electrical and Computer Engineering, University of Illinois at Chicago (UIC), IL, USA, since January 2009. From July 2007 until July 2008, she was a Lecturer at Harvard University. Her research focuses on multi-user information theory and applications to cognitive and software-defined radio, radar, and relay and two-way communication networks. She is an Editor of the IEEE Transactions on Wireless Communications and the IEEE Journal of Selected Areas of Communications (Cognitive Radio Series). She was a recipient of the NSF CAREER Award in 2011 and was named UIC's Researcher of the Year in the "Rising Star" category in 2012.

Daniela Tuninetti received the M.S. degree in telecommunication engineering from the Politecnico di Torino, Turin, Italy, in 1998 and the Ph.D. degree in electrical engineering from ENST/Telecom ParisTech, Paris, France (with work done at the Eurecom Institute, Sophia Antipolis, France), in 2002. From 2002 to 2004, she was a Postdoctoral Research Associate with the School of Communication and Computer Science, EPFL/Swiss Federal Institute of Technology in Lausanne, Lausanne, Switzerland. Since January 2005, she has been with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL, USA, where she currently is an Associate Professor. Her research interests are in the ultimate performance limits of wireless interference networks, with special emphasis on cognition and user cooperation. She was the Editor-in-Chief of the IEEE Information Theory Society Newsletter from 2006 to 2008, an Associate Editor of the IEEE Communication Letters from 2006 to 2009, and an Editor of the IEEE Transactions on Wireless Communications from 2009 to 2014. She currently is an Associate Editor of the IEEE Transactions on Information Theory. She regularly serves on the technical program committees of IEEE workshops and conferences. She was the Communication Theory Symposium Co-Chair of the 2010 IEEE International Conference on Communications (ICC 2010). She was a recipient of the Best Student Paper Award at the European Wireless Conference in 2002 and the NSF CAREER Award in 2007.