ECE 341: Probability and Random Processes for Engineers, Spring 2012
Quiz 1, 01.25.2012
Name:
Solutions

Do **one** of the following two problems (or both if you like!), see other side of page.

Problem 1. In an experiment, A, B, C, D are events with probabilities $P[A] = 1/4$, $P[B] = 1/8$, $P[C] = 5/8$ and $P[D] = 3/8$. Furthermore, A and B are disjoint, while C and D are independent.

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1. Find $P[A \cap B]$, $P[A \cup B]$, $P[A \cap B^c]$, $P[A \cup B^c]$.
2. (15) Are A and B independent?

10 10 10

3. Find $P[C \cap D]$, $P[C \cap D^c]$ and $P[C^c \cap D^c]$.
4. (15) Are C^c and D^c independent?

1. Since $A \cap B = \phi \Rightarrow P[A \cap B] = 0$.
Since $P[A \cap B] = 0 \Rightarrow P[A \cup B] = P[A] + P[B] - P[A \cap B] = 1/4 + 1/8 - 0 = 3/8$
Since $A \cap B^c = A$ when $A \cap B = \phi$, $P[A \cap B^c] = P[A] = 1/4$.
Then, $P[A \cup B^c] = P[B^c] = 1 - P[B] = 1 - 1/8 = 7/8$.

2. No, since $P[A \cap B] \neq P[A]\, P[B]$.

3. Since C, D are independent, by definition $P[C \cap D] = P[C]\, P[D] = 15/64$.
From Venn diagrams, notice that $P[C \cap D^c] = P[C] - P[C \cap D] = 25/64$
By de Morgan's law, $P[C^c \cap D^c] = P[(C \cup D)^c] = 1 - P[C \cup D] = 15/64$.

4. Yes, since $P[C^c \cap D^c] = P[C^c]\, P[D^c]$.

Problem 2. Shuffle a deck of 52 (13 different types, one of each of four suits. E.g. there are four aces (hearts, diamonds, spades, clubs)) cards and pick two cards at random. Observe the *sequence* of the two cards in the order in which they were chosen.

10 1. How many outcomes are in the sample space?

30 2. How many outcomes are in the event that the two cards are the same type but different suits?

20 3. What is the probability that the two cards are the same type but different suits?

40 4. Redo the three parts above if order is unimportant (*hint: this is easy!*).

1. Since order matters here, there are 52×51 outcomes.

2. 2 cards are of same type but different suits if, we pick any of the 52 cards 1st, then force the 2nd to be the same type. There are thus 52×3 outcomes.

3. This probability is simply the # of outcomes in 2 divided by the total number of outcomes, calculated in 1.

   Thus, $P[\text{same type, different suit}] = \frac{156}{2652} = \frac{1}{17}$

4. If order is unimportant then both 1. and 2. divide by 2 (as K♡, 8♡ = 8♡, K♡ and same for all other pairs, we counted them twice!)

   So 1. becomes $\binom{52}{2} = \frac{52 \times 51}{2}$

   2. becomes $\frac{52 \times 3}{2}$

   3. stays the same as the 2's cancel!