ECE 341: Probability and Random Processes for Engineers, Spring 2012
Quiz 2, 02.15.2012
Name:

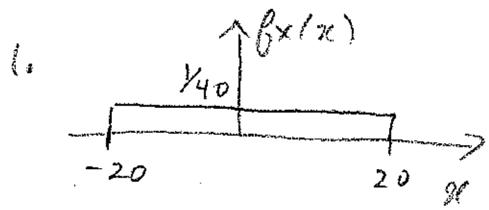
Instead of considering a quantizer as we did last class, let's consider an "amplitude limiter." X is the random variable input, and is uniform on $[-20, 20]$. This is input to the "amplitude limiter", whose output, a new random variable Y, relates to the input X as follows:

$$Y = \begin{cases} -5 & X \leq -5 \\ X, & X \in [-5, 5] \\ 5, & X \geq 5 \end{cases}$$

1. Plot the pdf of X and Y.
2. Find $E[X]$ and $E[Y]$.
3. Find $E[X^2]$ and $E[Y^2]$.

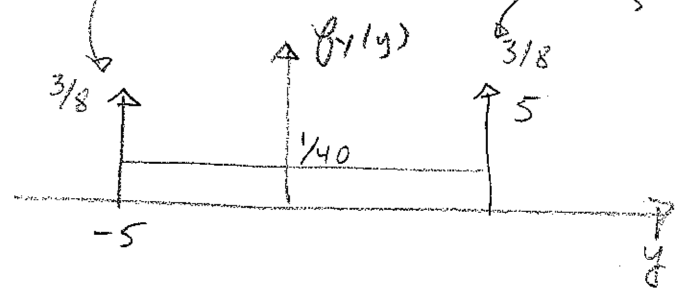
Solution:

1.



Mass is $\int_{-20}^{-5} \frac{1}{40}\,dx = \frac{15}{40} = \frac{3}{8}$

Mass is $\int_{5}^{20} \frac{1}{40}\,dx = \frac{3}{8}$



2. $E[X] = 0$ by symmetry (or as we know mean of a uniform RV).

$E[Y] = 0$ by symmetry.

3. $E[X^2] = \frac{(20-(-20))^2}{12} = \frac{(40)^2}{12} = \frac{1600}{12} = \cancel{\frac{400}{3}}$

$$E[Y^2] = \int_{-5}^{5} y^2 f_Y(y)\,dy = \int_{-5}^{5} y^2 \left(\frac{3}{8}\delta(y+5) + \frac{3}{8}\delta(y-5) + \frac{1}{40}\right) dy$$

$$= \frac{3}{8}(-5)^2 + \frac{3}{8}(5)^2 + \frac{1}{40}\frac{y^3}{3}\Big|_{-5}^{5}$$

$$= \frac{150}{8} + \frac{1}{40}\left[\frac{125}{3} - \frac{(-125)}{3}\right] = \frac{150}{8} + \frac{250}{120}$$