Instead of considering a quantizer as we did last class, let's consider an “amplitude limiter.” $X$ is the random variable input, and is uniform on $[-20, 20]$. This is input to the “amplitude limiter”, whose output, a new random variable $Y$, relates to the input $X$ as follows:

$$
Y = \begin{cases} 
-5 & X \leq -5 \\
X & X \in [-5, 5] \\
5 & X \geq 5
\end{cases}
$$

1. Plot the pdf of $X$ and $Y$.
2. Find $E[X]$ and $E[Y]$.

Solution:

1. 

2. $E[X] = 0$ by symmetry (or as we know mean of a uniform RV).

3. $E[X^2] = \left(\frac{20 - (-20)}{12}\right)^2 = \left(\frac{40}{12}\right)^2 = \frac{1600}{144} = \frac{125}{9}$

   $$
   E[Y^2] = \int_{-5}^{5} y^2 f_Y(y)\,dy = \int_{-5}^{5} y^2 \left(\frac{3}{8} \delta(y - 5) + \frac{3}{8} \delta(y + 5) + \frac{1}{40}\right)\,dy
   $$

   $$
   = \frac{3}{8} (-5)^2 + \frac{3}{8} (5)^2 + \frac{1}{40} \left(\frac{4}{3}\right)^3 \left|_{-5}^{5}\right.
   $$

   $$
   = \frac{150}{8} + \frac{1}{40} \left[\frac{125}{3} - \frac{(-125)}{3}\right] = \frac{150}{8} + \frac{25\sqrt{2}}{12}
   $$