

ECE 341: Probability and Random Processes for Engineers, Spring 2012

Quiz 3, 03.02.2012

Name: Solutions

We monitor calls and we classify a call as either a voice call (V) when someone is speaking, or a data call (D) when the call is carrying a fax or modem signal. We further classify the call as either short (S) or long (L) depending on whether the call lasts less than, or more than three minutes. Based on the data collected by the telephone company, we use the following probability model: $P[V] = 0.8, P[L] = 0.6, P[D \cap S] = 0.1$. Find the following:

- 10 1. $P[V \cap L]$
- 10 2. $P[D \cap L]$
- 10 3. $P[S \cup L]$
- 10 4. $P[V \cap D]$
- 15 5. $P[V|L]$
- 15 6. $P[D|V]$
- 15 7. Two events A and B are called independent if $P[A \cap B] = P[A]P[B]$, or equivalently if $P[A|B] = P[A]$. Are the events V (voice) and L (long) independent? Justify your answer.
- 15 8. Find a lower bound on the expected duration of a call (in minutes).

Solution:

Find joint PDF of Call type + duration

		Duration	
		S	L
Type	V	0.3	0.5
	D	0.1	0.1

↗ given

1. $P[V \cap L] = 0.5$
2. $P[D \cap L] = 0.1$
3. $P[S \cup L] = 1$
4. $P[V \cap D] = 0$
5. $P[V|L] = \frac{0.5}{0.6} = \frac{P(V,L)}{P(L)} = \frac{0.5}{0.6}$
6. $P[D|V] = 0$ as $P[D, V] = 0$

7. $P[V] = 0.8, P[L] = 0.6$
 $P[V \cap L] = 0.5 \neq P[V]P[L] = 0.48$
 So, dependent.

8. $E[\text{duration}] = P[S] E[\text{duration}|S] + P[L] E[\text{duration}|L]$
 $\geq P[S] \cdot 0 + 0.6(3) = \underline{\underline{0.8 \text{ minutes}}}$