ECE 341: Probability and Random Processes for Engineers, Spring 2012
Quiz 5, 04.18.2012
Name:

Solution

$X_n$ is a random sequence with mean $\mu[n] = E[X_n]$ and auto-correlation function $R_X[n, k]$ (we know both of these). We make noisy observations of this random sequence, i.e. observe $Y_n = X_n + N_n$, where $N_n$ is a random noise sequence with $\mu_N = E[N_n] = 0$, and autocorrelation $R_N[n, k]$. Assume that the noise sequence $N_n$ is independent of $X_n$.

1. Find $E[Y_n]$.

20. Find the auto-correlation function of $Y_n$, $R_Y[n, k]$.

20. Find the auto-covariance function of $Y_n$, $C_Y[n, k]$.

Solution:


2. $R_Y[n, k] = E[Y_n Y_{n+k}] = E[(X_n + N_n)(X_{n+k} + N_{n+k})]$

   $= E[X_n X_{n+k}] + E[X_n N_{n+k}] + E[N_n X_{n+k}] + E[N_n N_{n+k}]$

   $= R_X[n, k] + E[X_n]E[N_{n+k}] + E[N_n]E[X_{n+k}] + R_N[n, k]$

   $= R_X[n, k] + 0 + 0 + R_N[n, k]$

3. $C_Y[n, k] = E[(Y_n - E[Y_n])(Y_{n+k} - E[Y_{n+k}])]

   = E[Y_n Y_{n+k}] - E[Y_n]E[Y_{n+k}] - E[Y_{n+k}]E[Y_n] + E[E(Y_n)E[Y_{n+k}]]$

   $= R_Y[n, k] - \mu[n]\mu[n+k] - \mu[n]\mu[n+k] + \mu[n]\mu[n+k]$

   $= R_Y[n, k] - \mu[n+k]\mu[n] - \mu[n]\mu[n+k] + \mu[n]\mu[n+k]$

   $= R_Y[n, k] - \mu[n+k]\mu[n]$. 