Sums of RVs example (requested by Tejumade)

Telephone calls are classified as either voice (V) or data (D), and we know that for a given call $P[V] = 0.8$, $P[D] = 0.2$. Data and voice calls occur independently of one another. Let K_n = # data calls out of n calls.

1. What is $E[K_n]$?
2. What is $Var[K_n]$?
3. Use the central limit theorem to estimate $P[K_{100} \geq 18]$.

Solution:

1. Define indicator RVs i.e. let

$$D_i = \begin{cases} 1 & \text{if call is Data} \\ 0 & \text{if call is voice} \end{cases} \quad \text{for } i = 1, 2, \ldots, n$$

Then

$$\boxed{K_n = D_1 + D_2 + D_3 + \cdots + D_n.}$$

PMF of D_i ($\forall i$) is

$$P_D(d) = \begin{cases} 0.8 & d = 0 \\ 0.2 & d = 1 \end{cases}$$

p.

$E[D_i] = 0.2 = p = 0.8(0) + 0.2(1)$.

$Var[D_i] = 0.2 \cdot 1^2 - (0.2)^2 = 0.16$

$= E[D_i^2] - (E[D_i])^2$

$= p(1-p)$.

(1)

$E[K_n] = E[D_1 + D_2 + \cdots + D_n] = E[D_1] + E[D_2] + \cdots + E[D_n]$.

$= n(0.2)$.

(2)

as IID

$Var[K_n] = \sum_{i=1}^{n} Var[D_i] = n(0.16)$

(3) $P[K_{100} \geq 18] = 1 - P[K_{100} \leq 18]$

FORM OF A CDF. SO CAN APPLY CLT!! pg. 259.

$$\approx 1 - \Phi\left(\frac{18 - 100(0.2)}{\sqrt{100(0.16)}}\right)$$

$E[K_{100}]$

$std(K_{100}) = \sqrt{Var(K_{100})}$

this looks like a Gaussian as n gets large!

$$= 1 - \Phi\left(-\frac{1}{2}\right) = \Phi(1/2) = 0.6915$$

look up in table!