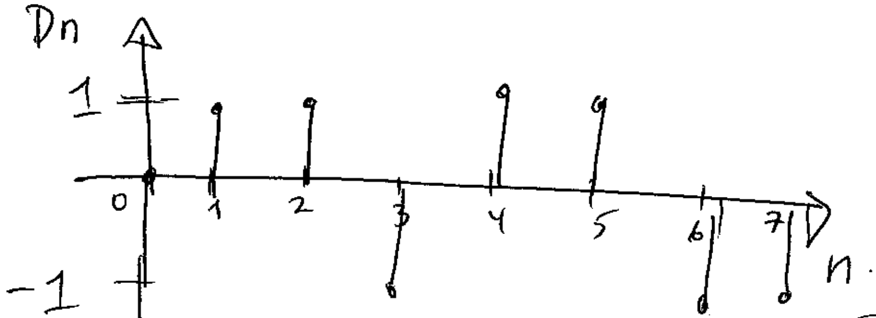
EX: Random walk:

A particle moves along a line in time steps, ie. at time n it either moves left 1 or ~~of~~ right 1 according to the outcome of a Bernoulli RV I_n (at time n). Let D_n denote the motion of the particle at time n, where $D_0 = 0$. Then $(n \geq 1)$.

$$D_n = \left.\begin{cases} +1 & \text{if } I_n = 1 \\ -1 & \text{if } I_n = 0 \end{cases}\right\} D_n = 2I_n - 1$$

- ~~Show~~ Draw a sample function / sequence of D_n



$P[I_n = 0] = 1-p$
$P[I_n = 1] = p$.

- Find $E[D_n]$, $\text{var}[D_n]$

$$E[D_n] = E[2I_n - 1] = 2\underbrace{E[I_n]}_{\text{Bernoulli}(p)} - \overbrace{E[1]}^{1} = 2p - 1.$$

$$\text{Var}[D_n] = \text{Var}(2I_n - 1) = \underbrace{4}_{2^2}\text{Var}(I_n) = 4\left[E[I_n^2] - (E[I_n])^2\right]$$

$$= 4\left[p - p^2\right] = 4p(1-p).$$

- What is the position of the particle at time n? Let's call this S_n...

$$S_n = \sum_{i=1}^{n} D_i$$

→ can calculate all sorts of things about S_n ....

Ch.11 Filtering a random process (11.1, 11.2, 11.8)

$X(t) \longrightarrow$ [Linear time invariant system] $\longrightarrow$ What comes out?

random process — $h(t)$ — $\underbrace{}$ not random! — Another random process! $Y(t)$

What does this outcome RP, $Y(t)$ look like?

Theorem 11.2: If $X(t)$ is a WSS random process, and $X(t)$ is input to a linear time invariant (LTI) filter with impulse response $h(t)$, then the output $Y(t)$ has the properties:

(a) $Y(t)$ is also WSS. with:

$$\mu_Y = E[Y(t)] = \mu_X \int_{-\infty}^{+\infty} h(u)\, du.$$

$$R_Y(\tau) = E[Y(t)Y(t+\tau)] = \int_{-\infty}^{+\infty} h(u) \int_{-\infty}^{+\infty} h(v)\, R_X(\tau + u - v)\, dv\, du.$$

(b) $X(t)$, $Y(t)$ are jointly WSS with.

$$R_{XY}(\tau) = \int_{-\infty}^{+\infty} h(u)\, R_X(\tau - u)\, du. = E[X(t)Y(t+\tau)].$$

(c) Output autocorrelation function satisfies:

$$R_Y(\tau) = \int_{-\infty}^{+\infty} h(-w)\, R_{XY}(\tau - w)\, dw. = E[Y(t)Y(t+\tau)]$$

EX: $X(t)$ is WSS with $\mu_X = 10$ volts is input to an LTI filter with impulse response

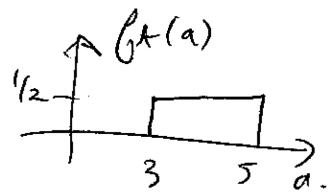
$$X(t) \rightarrow \boxed{h(t)} \rightarrow Y(t).$$

$$h(t) = \begin{cases} e^{t/0.2} & 0 \leq t \leq 0.1 \quad \text{(seconds)} \\ 0 & \text{else.} \end{cases}$$

What is $E[Y(t)]$? $= \mu_Y = \mu_X \int_{-\infty}^{+\infty} h(u)\,du = 10 \int_{0}^{0.1} e^{u/0.2}\,du = 2(e^{0.5} - 1)$

EX: Let $X(t) = A\sin(w_0 t)$ where w_0 is constant, A is a RV that is continuous and uniform over $[3,5]$.

- What is the pdf of A? $f_A(a) = \begin{cases} \frac{1}{2} & \text{for } a \in [3,5] \\ 0 & \text{else.} \end{cases}$



- Is $X(t)$ WSS? To check: 1) Is $E[X(t)] = \mu_X$ constant?
  2) Is $R_X(t,\tau)$ a function only of τ?

$$E[X(t)] = E[A \sin(w_0 t)] = E[A] \cdot \sin(w_0 t) = 4 \sin(w_0 t).$$

$\rightarrow$ this is a function of time and hence not WSS!!

- Suppose w_0 is also random, w_0 is independent of A and w_0 is uniform on $[0, 2\pi]$. $f_{w_0}(w_0) = \begin{cases} \frac{1}{2\pi} & w_0 \in [0, 2\pi] \\ 0 & \text{else} \end{cases}$

~~What is $E[X(t)]$? =~~

$f_{A,W_0}(a, w_0) = f_A(a)\, f_{W_0}(w_0)$ as A, W_0 are independ. 3

$$= \begin{cases} \frac{1}{2} \cdot \frac{1}{2\pi} & \text{for } a \in [3,5],\ w_0 \in [0, 2\pi] \\ 0 & \text{else.} \end{cases}$$

So, $E[Y(t)] = E[A \sin(W_0 t)] = E[A]\, E[\sin(W_0 t)]$.

$$= 4 \cdot \int_0^{2\pi} \sin(w_0 t)\, dw_0 = -4 \cos(w_0 t) \Big|_0^{2\pi}$$

$$= 4 \cdot 0 = 0$$