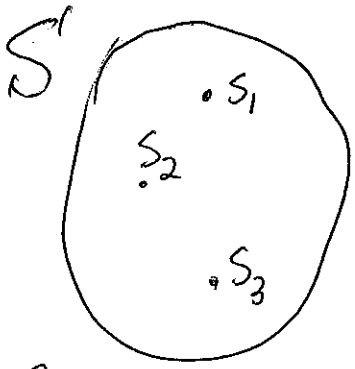
Ch. 10 Stochastic Processes



Sample space corresponding to an experiment.

Random variable

- ~~maps~~ associates a number to each outcome s in sample space S

Notation: X $(X(s))$ (e.g. roll a die)

Random vector

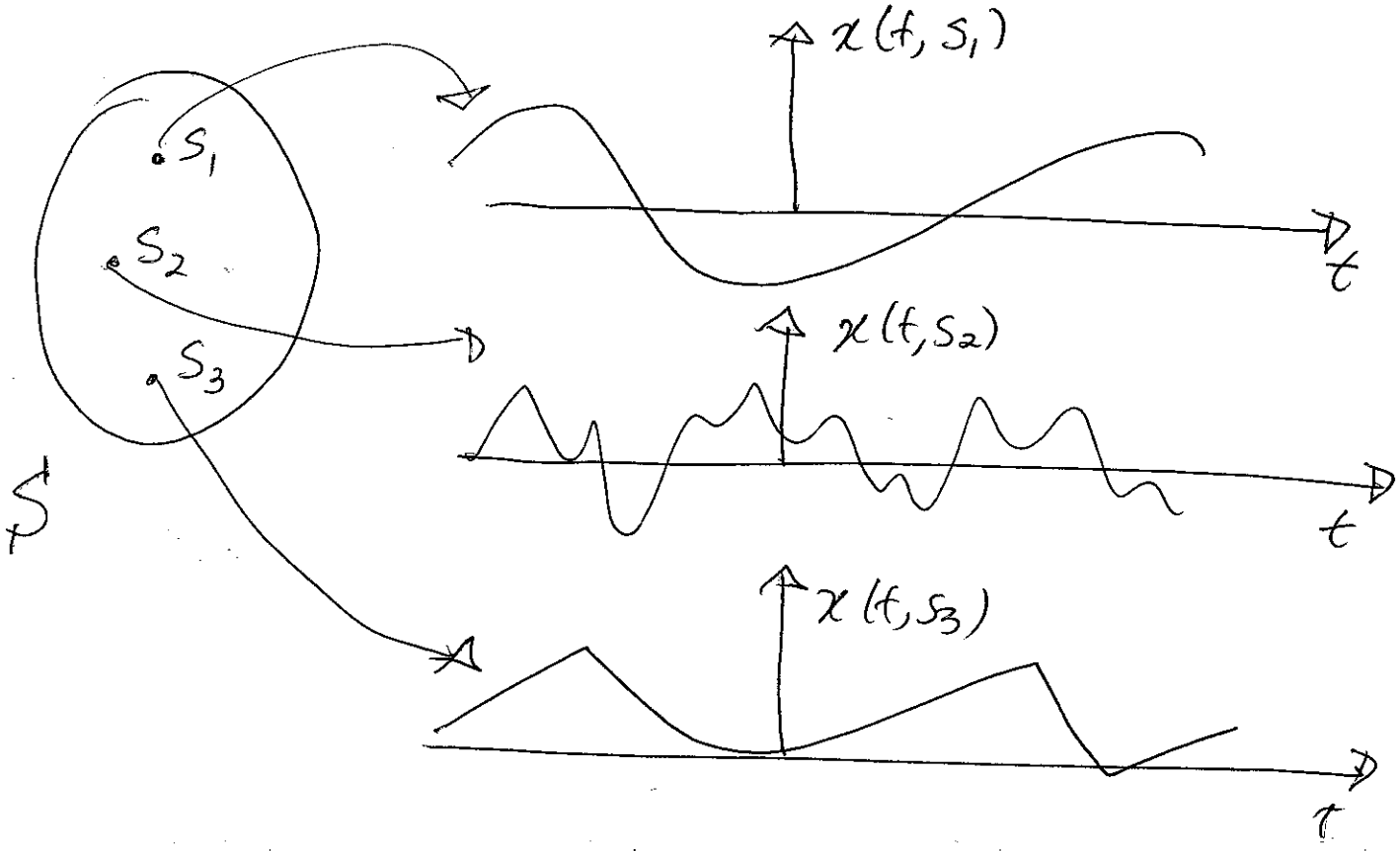
- associates a vector to each outcome $s \in S$

Notation: $[X\ Y]$ or $[X, X_2, \ldots X_n]$. (e.g. roll 2 die)

Random/stochastic process

- associates a sample function to each outcomes $s \in S$

(e.g. roll a die over + over + over. . . .)



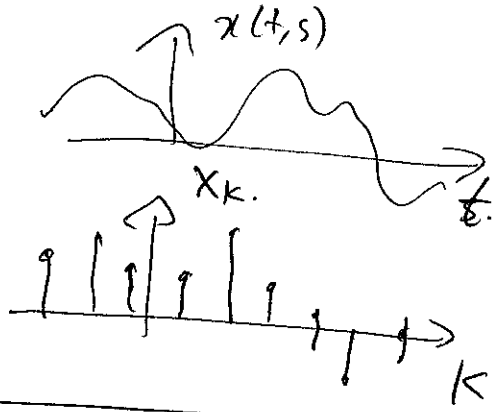
Definition: a sample function $x(t, s)$ is the time function associated with an outcome $s \in S$.

Definition: The ensemble of a RP is the set of all sample functions.

2 Types of Random processes:

1) Random waveforms: denoted as $X(t)$
(analog waveforms)

2) Random sequences: denoted as X_k
(sampled waveforms).

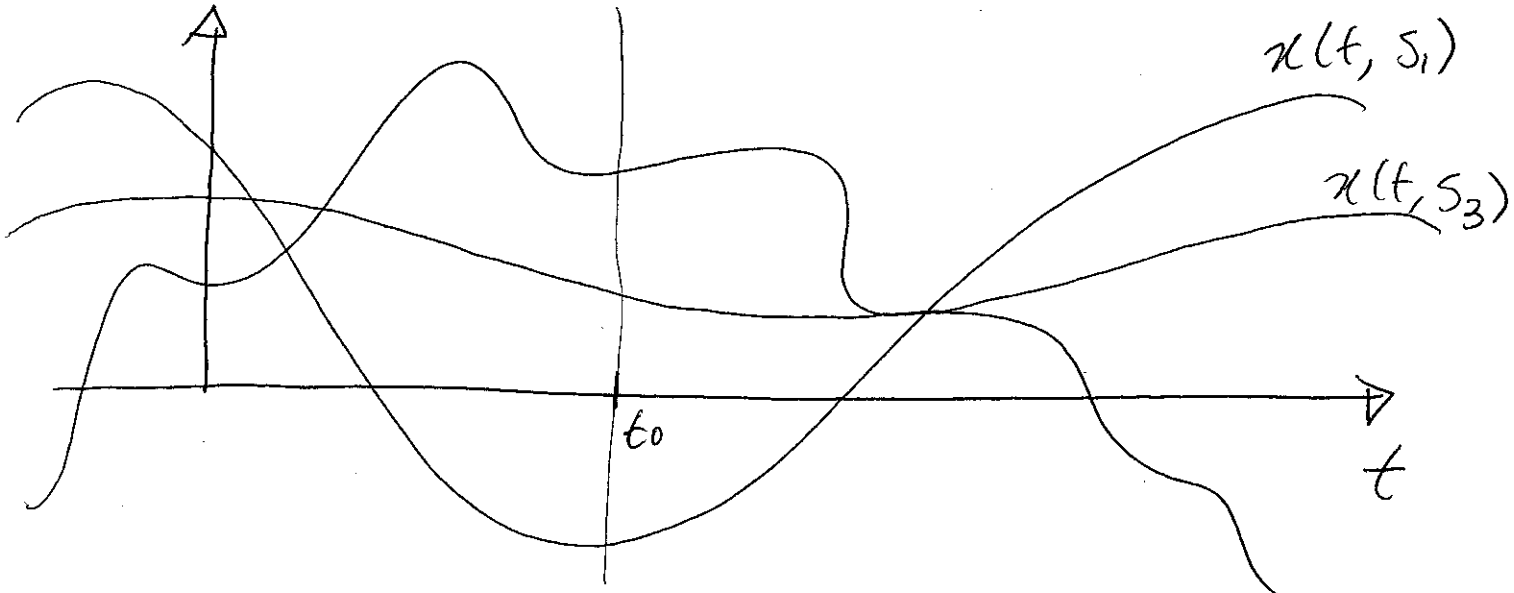


(t in seconds)

EX: Starting at launch time $t=0$, let $X(t)$ denote the temp-on the surface of a space shuttle. With each launch s, record the temperature sequence $x(t, s)$.

- What is the ensemble? all possible temperature waveforms that we might observe.
- What does $x(8073.68, 175) = 207$ mean?
  On the 175th lauch, at time 8073.68 seconds, the recorded temperature was 207 degrees.

Relationship between random processes, random vectors, and random variables? YES!



- For a given $t_0 \Rightarrow X(t_0, s) \equiv X(t_0)$ is a <u>random variable</u>!
  (notation equivalence)

- For given $t_1, t_2 \Rightarrow [X(t_1, s)\ X(t_2, s)] \equiv [X(t_1)\ X(t_2)]$ is a <u>pair of random variables.</u>

- For any $t_1, t_2, \ldots, t_n \Rightarrow [X(t_1)\ X(t_2) \cdots X(t_n)]$ is a <u>random vector</u>

Averages:

- ensemble average: Fix t_0. $\Rightarrow X(t_0)$ is then a RV

  $\Rightarrow E[X(t_0)]$ is an "ensemble average" (average over s)

- time average: Fix outcome $s_0 \Rightarrow x(t, s_0)$ is a deterministic function (not random, known!)

  $\Rightarrow$ average of $x(t, s_0) = \int_{-\infty}^{+\infty} x(t, s_0)\, dt$

  is a "time average" (average over t)

EX: Continuing the launch example, what is (time or ensemble average)

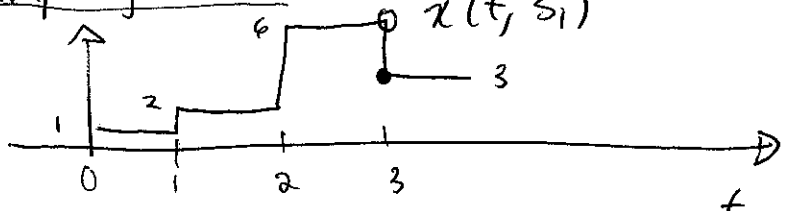
"average temp. of the 175th launch"? time average (fixed s!)

"average temp. ~~of~~ at 100 seconds"? ensemble average (fixed t!)

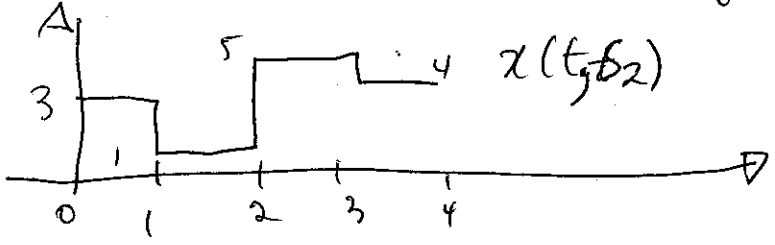
EX: At time instant $T = 0, 1, 2, \ldots$ roll a die and record the outcome N_T, where $1 \le N_T \le 6$ (integer). Define the random process $X(t)$ such that: for $T \le t < T+1$, $X(t) = N_T$.

- Draw some sample functions.

$s_1 = 1, 2, 6, 3$



$s_2 = 3, 1, 5, 4$



- What is $X(3.5)$? Is a RV equal to the roll at time $T = 3$.

- What is the distribution of $X(3.5)$?

$$P_{X(3.5)}(x) = \begin{cases} 1/6 & x \in [1, \ldots 6] \\ 0 & \text{else} \end{cases}$$