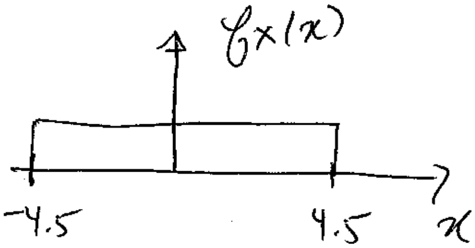
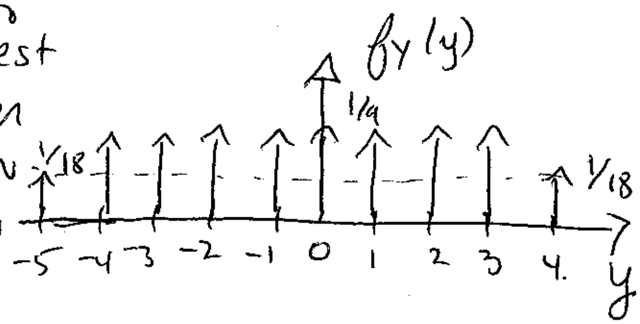
Ex: Quantization

continuous.

discrete/digital.

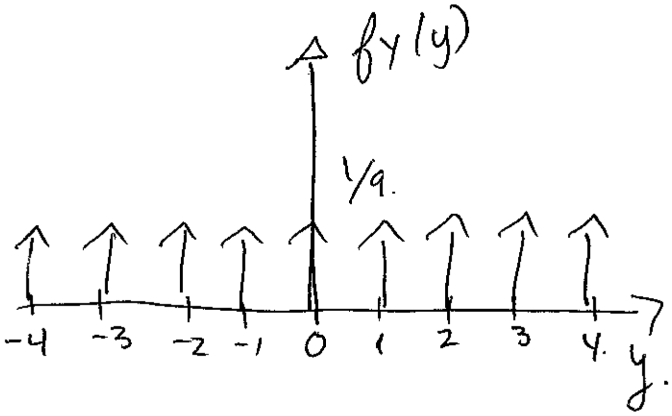


$\rightarrow Q_1 \rightarrow$ maps to nearest integer below "floor" $\lfloor X \rfloor$



map to nearest integer.

Q_2



What is $E[Y]$?

What is $E[Y^2]$?

What is $E[(X-Y)^2]$?

For Q_1: $E[Y] = \frac{1}{18}(-5) + \frac{1}{9}(-4 + \cancel{-3} + \cancel{-2} + \cancel{-1} + 0 + \cancel{1} + \cancel{2} + \cancel{3}) + \frac{1}{18}\cdot 4.$

$$= \frac{-5}{18} - \frac{8}{18} + \frac{4}{18} = \frac{-9}{18} = -\frac{1}{2}.$$

For Q2: $E[Y] = 0$ by symmetry.

$$= \frac{1}{9}(-4 + -3 + -2 + -1 + 0 + 1 + 2 + 3 + 4).$$

$$= \sum_{y \in S_Y} y \cdot f_Y(y).$$

<u>For: Q1</u>: $E[Y^2] = \sum_{y \in S_Y} y^2 p(y)$

$$= \frac{25}{18} + \frac{16}{9} + \frac{9}{9} + \frac{4}{9} + \frac{1}{9} + \frac{0}{9} + \frac{1}{9} + \frac{4}{9} + \frac{9}{9} + \frac{16}{18}.$$

$$= 7.1\overline{6}$$

$$E[X^2] = \frac{(4.5-(-4.5))^2}{12} = \frac{81}{12} = 6.75.$$

$E[(X-Y)^2]$. = the "mean-squared error" of this quantizer

$$= E[X^2 - 2XY + Y^2] = E[X^2] - 2E[XY] + E[Y^2].$$

What is this? Ch. 4

Try a different approach!

Consider the case when, for Q1,

→ $-4.5 \le X < -4$ then $Y = -5$ } $(X-Y)^2 = (X-(-5))^2$

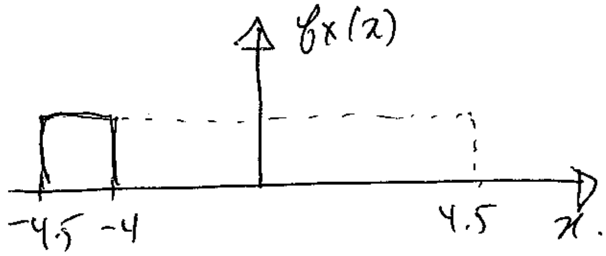
→ $-4 \le X < -3$ then $Y = -4$ } $(X-Y)^2 = (X-(-4))^2$

⋮ ⋮ ⋮

→ $3 \le X < 4$ Then $Y = 3$

→ $4 \le X \le 4.5$ then $Y = 4$. } $(X-Y)^2 = (X-4)^2$

Define $Z = X - Y$. Then $f_{Z|(-4.5 \le X < -4)}(z)$ is.



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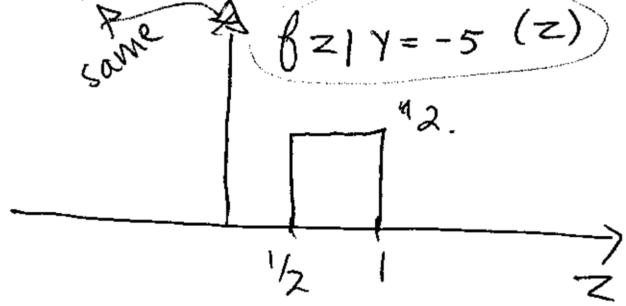
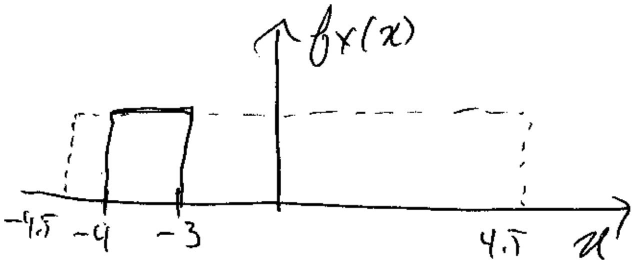
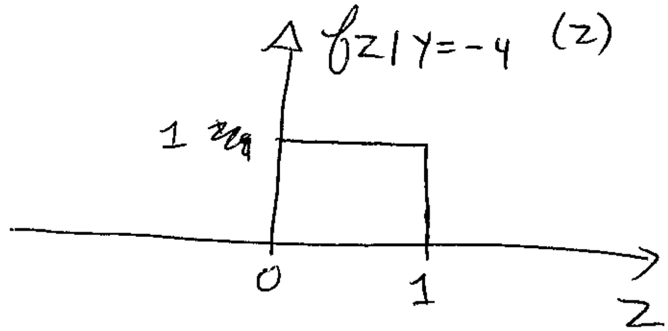


For a "middle case", $-4 \le X < -3 \Rightarrow Y = -4$.



$\Rightarrow$



$$E[(X-Y)^2] = E[Z^2].$$

$$= E[Z^2 \mid \overset{Y=-5}{-4.5 \le X < -4}]\, P[\overset{Y=-5}{-4.5 \le X \le -4}]$$

$$+ E[Z^2 \mid -4 \le X < -3]\, P[-4 \le X < -3]$$

$$+ \dots$$

$$+ E[Z^2 \mid 3 \le X < 4]\, P[3 \le X \le 4]$$

$$+ E[Z^2 \mid 4 \le X \le 4.5]\, P[4 \le X \le 4.5].$$

$$= \frac{1}{18} \int z^2 f_{Z|Y=-5}(z)\, dz$$

$$\left.\begin{aligned} &+ \frac{1}{9} \int z^2 f_{Z|Y=-4}(z)\, dz \\ &+ \dots \\ &+ \frac{1}{9} \int z^2 f_{Z|Y=3}(z)\, dz \end{aligned}\right] \text{middle.}$$

$$+ \frac{1}{18} \int z^2 f_{Z|Y=4}(z)\, dz.$$

$$= \frac{1}{18}\, 2 \int_{1/2}^{1} z^2\, dz + \frac{1}{9} \int_0^1 z^2\, dz + \frac{1}{9} \int_0^1 z^2\, dz + \dots + \frac{2}{18} \int_0^{1/2} z^2\, dz.$$

In the previous example, we used 2 concepts:

1. Functions of continuous RVs (quantizer)
2. Conditioning a continuous RV